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Does public debt secure social peace? A diversionary theory of public debt management

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Abstract

This study analyzes the strategic use of public debt. Contrary to the classical view that politicians can use public debt to tie the hands of their successors, we show that an incumbent government can take advantage of having tied its own hands before the election with the help of public debt. By doing this, it reduces the basis for future social conflicts and benefits from social peace during its term, which may enhance its chances of being re-elected. In addition, in the case of foreign or external public debt, the incumbent can strategically divert future social conflicts toward a common enemy (foreign creditors). Thus, by increasing public debt before the election, the incumbent can strengthen social cohesion during his/her mandate, both by reducing the basis of internal conflicts and by diverting citizens from internal toward external rent-seeking activities.

Keywords: Public debt, Election, Conflict, Rent-seeking.

1. Introduction

Traditional political budget cycle analysis suggests that election cycles in public spending and taxes can result from the strategic behavior of incumbent politicians who seek re-election. Opportunistic governments undertake pre-electoral expansionary fiscal policies because short-sighted electors appreciate low taxes and high expenditure (Nordhaus, 1975), or, in modern probabilistic voting models (see, e.g., Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) because rational forward-looking electors face informational problems. In this literature, voters are imperfectly informed about candidates' abilities or the environment, and a fiscal expansion that boosts economic activity may signal the incumbent's competencies. The latter is then tempted to stimulate economic activity to be viewed as the most competent candidate (see Rogoff and Sibert, 1988).

A major shortcoming of this approach is that pre-electoral public deficits give rise to increases in public debt, and it is difficult to believe that such increases signal politicians'

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competence. On the contrary, forward-looking electors are aware of the intertemporal government budget constraint and know that post-electoral adjustments will follow pre-election fiscal expansions.

By observing large public debt issuance before the election, these electors may consider, by contrast, that the incumbent is relatively incompetent at managing public finances and should not be rewarded, perceiving incumbents' actions as electoral fiscal manipulations. Consequently, pre-electoral increases in deficits can damage rather than improve incumbents' reputation if voters are "fiscally conservative" (Peltzman, 1992).[2]

In this study, we propose a new reason why an incumbent increases public debt before the election. We state that a large debt burden generated by the incumbent and distributed across social groups can prevent social conflicts during his/her future mandate. A considerable number of examples show that favorable economic conditions, generating windfall gains for public finance, are likely to generate claims to increase public spending or cut taxes, to the benefit of certain social groups or lobbies defending particular interests. By issuing public debt before the election, the incumbent can credibly commit to future austerity measures that will discipline his/her constituents during his/her term, thus avoiding such rent-seeking activities. In other words, pre-electoral increases in public debt will secure future social peace because citizens know that the elected government will not be able to handle all claims.[3]

Post-electoral austerity measures can be a source of dissatisfaction that undermines the popularity of the elected government. However, in the case of external public debt (e.g., federal or foreign debt), a strategic government may attempt to establish internal cohesion by creating a common enemy (creditors in the federation or abroad). By di-

[2]In the United States, for example, Peltzman (1992) shows that electors punish politicians who let public spending increase. The explanation is related to progressive fiscal systems: on average, voters are wealthier than non-voters and penalize spender incumbents because they will pay the price of future fiscal adjustments. Such fiscal "conservatism" of electors is also found by Bertola and Drazen (1993) and Brender and Drazen (2008), who show, using a sample of 74 countries from 1960 to 2003, that pre-election deficits reduce the chances of the incumbent being re-elected. Garmann (2017) shows that if voters are highly fiscally conservative, incumbents can even decrease spending before elections.

[3]The intuition that debt can be used to manage social conflicts is not new. Capital structure theories (see, e.g., Grossman and Hart, 1982; Jensen, 1986; Stulz, 1990, among others) emphasize the role played by corporate debt in reducing agency conflicts among workers, managers, and shareholders. Specifically, by increasing the probability of failure, private debt exerts a threat that can discipline workers by forcing them to accept low wages and poor working conditions. In this study, we show that the same type of argument can apply to public debt, which can be used as a disciplining device to limit rent-seeking behavior during the term.

verting the discontent caused by austerity policies toward outside scapegoats, it can thus avoid unpopularity. Public debt management can then serve to control future internal and external social conflicts for electoral purposes.

To develop these ideas, we build an original probabilistic voting setup with rent-seeking activities.

Our model is based on a two-period game between N districts that are part of a federal union. In our stylized setup, a district can describe any local authority such as a municipality, federated state or country belonging to a wider group; it can also be a region, federal state or supra-national union. In the first period, in each district, the local government provides a flow of public goods that can be financed using federal debt, namely, the debt issued by the federal authority in the first period (from a particular district point of view, public debt is then assimilable to foreign debt). At the end of the first period, an election is held in each district, which opposes an incumbent and a challenger. In the second period, the newly elected government will have to repay the debt burden, which reduces the resources available to finance the public good. However, there is a conflict between districts about debt repayment (a particular district can formally or informally default, negotiate to alleviate debt, and so on, in which case the other districts will have to pay the bill).

Consequently, there are two types of conflicts: an internal conflict among the citizens of a district (to capture the highest share of the local public good) and an external conflict under which other districts oppose this district (to avoid debt repayment). In the symmetric equilibrium, these two conflicts generate unproductive rent-seeking efforts that reduce the taxable income and thus the level of public good.

Our main finding is that through public debt, a strategic incumbent can manage these conflicts for his/her benefit. By increasing public debt today, he/she generates a debt burden tomorrow that reduces internal conflicts (i.e., the size of the feasible public good decreases) but strengthens external conflicts (to avoid repaying this burden).

Related literature. Our model is at the interface of two strands of the literature.

(i) On the one hand, our results can be compared with the so-called diversionary theory of war, whereby *"it appears to be a general law that human groups react to ex-*

ternal pressure by increased internal coherence" (Dahrendorf 1964, cited by Levine and Campbell 1972, p.31). According to Levy (1989), a diversionary war occurs when a leader begins a foreign conflict to divert attention from an ongoing domestic issue. In our model, by increasing debt, the incumbent creates a common enemy that serves to strengthen internal cohesion and limit social conflicts about the repartition of public goods.[4]

(ii) On the other hand, our model belongs to the abundant literature on the strategic use of public debt. Building on the pioneering works of Alesina and Tabellini (1990) and Persson and Svensson (1989), a number of studies have developed and tested the idea that if a government expects to be defeated, it will try to use debt strategically to constraint the future policies of his/her opponent.[5] By bequeathing a high debt burden to his/her successor, the incumbent can force his/her newly elected challenger to pay the bill and prevent him/her from carrying out his/her own policies.[6] In all this literature, public debt thus serves to tie the hands of possible successors.

Our model extends and challenges these results in three directions.

First, we propose a complementary point of view that suggests that the incumbent can benefit from tying his/her own hands. By doing this, he/she can credibly commit to not meet the claims of citizens during his/her term, discouraging rent-seeking activities. Therefore, in our model, public debt will be used strategically even in the absence of partisan preferences (as in Alesina and Tabellini, 1990), or disagreement between politicians about the desired level or composition of public goods (as in Persson and Svensson, 1989). Effectively, it is in the incumbent's interest to constrain his/her own room for maneuver rather than his/her challenger's one.

[4]This cohesion effect has been raised by a large number of social psychology and anthropology studies. For example, Sumner (1906) first introduced the term *ethnocentrism* and developed the idea that *"the exigencies of war with outsiders are what make peace inside. (...) These exigencies also make government and law in the in-group, in order to prevent quarrels and enforce discipline."* Such a concept is introduced in a formal model of rent-seeking by Münster and Staal (2011). Their study provides one of the first economic models with simultaneous conflicts at different levels (see also Katz et al., 1990). However, Münster and Staal (2011) use a static setup and do not study electoral competition, while our model is dynamic and mixes the rent-seeking and probabilistic voting approaches. Moreover, in their model, there is conflict either between or within groups, but not both at the same time (in the basic case in which all players decide simultaneously and independently how to allocate their resources), while both conflicts can appear simultaneously in our setup. This is an important point because by using public debt, an incumbent can move the weight of internal and external conflicts according to his/her preferences.

[5]See, for example, Pettersson-Lidbom (2001); Hodler (2011); Kirchgässner (2014).

[6]In the same vein, Aghion and Bolton (1990) present an interesting model in which the incumbent has an incentive to accumulate excessive public debt because he/she can more credibly commit not to default than his/her opponent.

Second, traditional works on the strategic use of public debt rest on the exogenous probability of re-election.[7] Yet, even if his/her chances are low, the incumbent may not undertake actions that reduce these chances, even if they strongly hinder the acts of his/her opponents.[8] In this way, a crucial feature of our model is that the inducement to public indebtedness is not related to the probability of losing the election. On the contrary, the incumbent strategically manages public debt to maximize the expected power rents he/she will receive, which positively depend on the probability of being re-elected.

Third, contrary to most of the literature, our model does not require that the incumbent has partisan preferences or benefits from a competence advantage. In our setup, holding power (for the incumbent) or being newly elected (for the challenger) is the only necessary ingredient to generate comparative advantages in managing social conflicts. In other words, the incumbent benefits (suffers) from comparative advantages (disadvantages) simply because he/she is in office, while the challenger is not.

The rest of the paper is organized as follows. Before exposing the theoretical model, Section 2 provides empirical evidence based on Canadian data. Section 3 presents the theoretical framework, Section 4 describes the political competition, and Sections 5 and 6 outline the solutions for the second-period and first-period equilibria, respectively. Section 7 presents our findings about public debt and social cohesion, while Section 8 is devoted to social welfare. Finally, Section 9 concludes.

2. Internal conflicts and public debt: Evidence from Canada

The main contribution of our model is that public debt can be strategically used by politicians for diversionary motives with two implications: (i) internal conflicts in a federal country are negatively linked to public debt and (ii) internal conflicts are strengthened if the incumbent is re-elected. In this section, we briefly expose some of the motivating evidence based on Canada data.

[7] In Alesina and Tabellini (1990), the incumbent has a chance of being re-elected, albeit his/her exact chances are uncertain; in Persson and Svensson (1989), the current government is certain to be removed.

[8] For example, Hodler (2011) finds that the incumbent never manipulates his/her opponent's public spending if he/she can ensure his/her own re-election.

Canada is suitable to illustrate our theory since its economic and parliamentary organization is consistent with our analytical framework. On the one hand, Canada is a federal state that divides government responsibilities between the federal government and 10 provinces in such a way that each province can finance a share of the public goods, while the federal government is responsible for funding the remaining share. On the other hand, the 10 provinces are considered to be sovereign in some areas (i.e., having their own governments, parliamentary elections, and internal constitutions), but they share responsibility with the federal government in other areas according to the Constitution Act from 1867. Such an economic and political organization affects how the power to spend money is shared, allowing many areas of spending to become vague, which may generate tension across provinces as well as between provinces and the federal government.

Data on internal conflicts are available from 10 Canadian provinces (Alberta, British Columbia, Manitoba, New Brunswick, Newfoundland, Nova Scotia, Ontario, Prince Edward Island, Quebec, and Saskatchewan) from the mid-1930s to 2013.[9] During our study period, we observed multiple violent events defined as internal conflicts at the provincial level, most in the largest provinces such that Alberta, British Columbia, Ontario, and Quebec (Quebec has the highest number of internal conflicts). Table A.5 lists the internal conflicts at the provincial level (see Appendix A).

Variable	Obs	Mean	Std.Dev.	Min	Max
Internal conflict	700	0.0514286	0.2210282	0	1
Provincial debt per revenue (%)	631	7.423291	5.408228	0.1910569	26.56126
Incumbents win	630	0.1746032	0.379929	0	1
Growth (%)	630	3.938864	3.586842	-5.201699	14.87949
Revenue	691	8647.783	17557.55	1	115911
Election year	700	0.3	0.4585852	0	1
Inflation (%)	680	3.655518	3.34598	-4.598511	14.36
Change in lntaxsize	630	0.0075569	0.0713626	-0.174653	0.32509
Time between elections	600	3.3	1.487847	1	6

Table 1: Summary Statistics

Table 1 shows that significant social events represent 5% of the available data, whereas elections in which incumbents win account for 17% of the cases, with large variations in

[9]We restrict our analysis to internal conflicts since external conflicts are not publicly available in Canada. The data on provincial internal conflicts come from the following sources (listed in Appendix A): "*A Brief History of Canada*", "*Riots and civil disorder in Canada*", "*List of conflicts in Canada*" and "*History Since Confederation*". The data on provincial debt and economic measures come from *Statistics Canada* and the data on elections come from the *Elections Canada* website.

economic growth levels, taxes, and inflation. As these variations may explain some of the dynamics of internal conflicts in Canadian provinces, we will use them in the conditional analysis.

2.1. Simple correlations

Using the variables described above, we examine the relationship between our variables of interest and the outcome from a simple linear correlation and a polychoric correlation.[10] We consider all the provinces first and then we exclude Quebec.[11] While simple linear correlations (Pearson correlations) are not fully informative for measuring the correlation between a discrete outcome variable and continuous ones, there is a slight negative correlation between provincial debt per revenue and internal conflicts (Table 2). This negative correlation becomes even stronger when Quebec is excluded.

Additionally, we find a slight positive correlation between the incumbents win variable and internal conflicts, which again strengthens when Quebec is excluded. The correlation effects also strengthen in the polychoric correlation between internal conflicts and provincial debt per revenue and between internal conflicts and the incumbents win variable.

Data Type of correlation Variable	All Data correlation internal conflict	All Data polychoric correlation internal conflict	No Quebec correlation internal conflict	No Quebec polychoric correlation internal conflict
Provincial debt per revenue	-0.0099	-0.0222	-0.0209	-0.0482
Incumbents win	0.0126	0.039	0.033	0.101
Lag growth	-0.0386	-0.078	-0.092	-0.204
Lag revenue	0.124	0.240	0.199	0.360
Election year	-0.012	-0.0344	-0.009	-0.0263
Inflation	-0.047	-0.1467	-0.0462	-0.1466
Change in ln of taxsize	0.0715	0.1402	0.0527	0.108
Time	0.048	0.1106	0.057	0.1339

Table 2: Correlations and Polychoric Correlations

This correlation information motivates us to pursue a more formal identification strategy to understand (i) the negative relationship between internal conflicts and public debt, and (ii) the positive relationship between internal conflicts and the incumbents win variable.

[10] To appropriately account for the relationship between a discrete and a continuous variable, we investigate the polychoric correlation of Drasgow (1986). The polychoric correlation measures the association between ordered or categorical variables and one or multiple continuous variables.

[11] Arguably, the events in Quebec are often related to issues of independence. This exclusion could also serve as a robustness check.

2.2. Identification Strategy

We use such events as riots and civil disorder with a significant social impact in Canadian provinces to measure of social instability (internal conflicts) in province i. The observed outcome variable is a dummy variable that takes 1 if a significant internal conflict happened in a province and 0 otherwise. The benchmark model is a reduced-form probability model of internal conflict (SU_{it}) in which the latent dependent variable is defined by

$$SU_{it} = \Pr(\alpha + \beta Debt_{it} + \gamma IW_{it} + \eta P_{it} + \psi El_{it} + \delta F_t) + u_{it}, \quad (1)$$

where $Debt_{it}$ is the annual provincial debt per federal GNP at time t, IW_{it} is a dummy for incumbents winning, El_{it} is a dummy for the year of an election, P_{it} represents provincial controls (revenue), F_t represents federal variables (economic growth, change in tax rates), and u_{it} are random logistic distributed errors.

The above relationships may suffer from endogeneity bias triggered by the relationship between debt and internal conflicts. To address this potential issue, we propose a correction of the bias using a control function (CF) approach, which is suitable for non-linear models.

Correction of Endogeneity Bias using the CF approach

The CF approach (see Heckman and Robb, 1985) corrects endogenous selection bias by modeling the endogeneity in the error term using a two-stage estimation procedure. The CF is a correction term needed as an additional regressor in the main equation of interest, as the outcome of interest is binary.[12] The procedure requires an exclusion restriction (or an instrument) to identify the impact of debt on internal conflicts. In the first stage, the endogenous variable is projected on the exclusion restriction(s) and a set of observed characteristics at the province level:

$$Debt_{it} = f(Z_{it}, X_{it}) + \epsilon_i, \quad (2)$$

where Z_{it} is the exclusion restriction(s), which is a variable correlated with the endogenous variable but not with the error in the main equation. Here, we consider Z_{it} to be the

[12] A standard instrumental variables procedure cannot be applied to discrete outcomes.

mortgage rate, and we also add its squared value to capture possible non-linearities of the mortgage rate on provincial debt. Therefore, we use two exclusion restrictions for the identification. These exclusion restrictions also satisfy the conditional independence assumption as in Abadie et al. (2002).[13] X_{it} is the set of controls used in the second stage (IW_{it},El_{it}, F_t).

The first-stage model can therefore be written as

$$Debt_{it} = \alpha_1 + \beta_1 Z_{it} + \beta_2 Z_{it}^2 + \gamma_1 IW_{it} + \psi_1 El_{it} + \delta_1 F_t + \epsilon_i. \tag{3}$$

Using the residuals from this first stage as the CF correction term in the second stage, the main equation of interest becomes

$$SU_{it} = \Pr(\alpha_2 + \beta_{1,2} Debt_{it} + \beta_{2,2} CF_{it} + \gamma_2 IW_{it} + \eta_2 P_{it} + \psi_2 El_{it} + \delta_2 F_t) + u_{it}, \tag{4}$$

where the benchmark model is augmented by CF_{it}, namely, the correction term obtained as the residuals from the first-stage estimation. Using this approach, we can correct the potential endogeneity between conflicts and debt.

Discussion of the results

First, we report the results of the first-stage model, where the endogenous variable (provincial debt per revenue) is modeled in function of the exclusion restriction (the mortgage rate and its square) and additional controls. Table 3 presents the results. The mortgage rate and its squared value are statistically significant. The residuals of the first-stage model are used as the CF in the second stage, which is our main specification of interest.

[13]The mortgage rate is not correlated with internal conflicts (correlation below 0.005), but is correlated with the debt variable (correlation about 0.2). Similar correlations are found with the squared mortgage rate variable. Consequently, the mortgage rate and its square can be used as exclusion restrictions (instrumental variables) to identify the probability of internal conflicts.

Variables	Provincial debt per revenue
Mortgage rate	-2.372**
	(1.003)
Mortgage rate squared	0.147**
	(0.0721)
Incumbents win	-0.436
	(0.537)
Lag growth	-0.113**
	(0.0465)
Election year	-0.736
	(0.450)
Inflation	-0.128**
	(0.0609)
Time	0.212***
	(0.0106)
Constant	10.28***
	(3.229)
Observations	522
Number of id	10
R-squared	0.525

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: First Stage: Panel Data Model of Provincial Debt per Revenue

We test the two hypotheses of interest using a probability model that applies to the benchmark model as well as the model that corrects the endogeneity of the provincial debt per revenue variable. We use two specifications of the logit model: the classical logistic probability model,[14] and a panel version of the logit model. Additionally, we provide the results when Quebec is excluded from the analysis.

Table 4 presents the results. The first column provides the benchmark model for all the data using the classical logistic specification, the second column presents the model that accounts for endogeneity using the CF in the logit specification, and the third column shows the panel version of the logit model with the CF correction. Columns four, five, and six are similar to columns one, two, and three, but we exclude Quebec data.

The results emphasize that internal conflict is negatively associated with provincial debt per revenue. They are robust to removing the data for Quebec and are even stronger in this case, as the correlation analysis suggests. When we control for the endogeneity of provincial debt per revenue using the CF approach, the negative effect is about five times larger in the parameter estimates, suggesting downward bias in the estimation of the

[14] We also test a penalized version of the logistic model that relates to rare events and the results remain unchanged.

benchmark model. This suggests an endogenous relationship between internal conflict and debt and that the proposed identification approach can capture this endogeneity. Finally, the positive association between incumbents winning and internal conflict is also validated by the model when we remove Quebec. Thus, even having controlled for endogeneity, the occurrence of internal social conflicts decreases with provincial public debt but may increase if the incumbent is re-elected. The rest of the paper develops a theory whose main implications are consistent with these two stylized facts.

Variables	(1) All Data Logit benchmark Internal conflict	(2) All Data Logit with CF Internal conflict	(3) All Data Panel logit with CF Internal conflict	(4) No Quebec Logit benchmark Internal conflict	(5) No Quebec Logit with CF Internal conflict	(6) No Quebec Panel logit with CF Internal conflict
Provincial debt per revenue	-0.114**	-0.650***	-0.535**	-0.162***	-0.701***	-0.645**
	(0.0514)	(0.159)	(0.226)	(0.0483)	(0.188)	(0.258)
CF		-36.99***	-33.53***		-33.98**	-34.64***
		(11.88)	(11.84)		(14.22)	(10.22)
Incumbents win	0.350	0.621	0.682	0.912	1.206	1.296***
	(0.732)	(0.736)	(0.584)	(0.927)	(0.883)	(0.318)
Lag growth	-0.136	-0.471***	-0.427**	-0.261***	-0.612***	-0.607***
	(0.0877)	(0.159)	(0.196)	(0.0816)	(0.149)	(0.133)
Lag reve	3.10e-05***	0.00024***	0.00019**	8.55e-05***	0.00027***	0.00025**
	(8.30e-06)	(6.04e-05)	(9.78e-05)	(1.84e-05)	(6.61e-05)	(0.00011)
Election year	-0.0783	1.223	1.043**	-0.237	1.281	1.184
	(0.711)	(0.796)	(0.515)	(0.943)	(0.983)	(0.750)
Inflation	-0.182	-0.617***	-0.572***	-0.186	-0.614***	-0.620***
	(0.113)	(0.174)	(0.0774)	(0.141)	(0.208)	(0.0687)
Change in ln taxsize	10.13***	42.51***	38.74***	11.52***	43.78***	43.47***
	(3.323)	(10.45)	(6.092)	(3.953)	(12.83)	(5.154)
Time	0.017	0.087***	0.080***	0.003	0.078**	0.0783***
	(0.0157)	(0.0262)	(0.0195)	(0.0175)	(0.0328)	(0.0205)
Constant	-2.302***	1.008	0.0657	-1.788***	1.165	0.729
	(0.634)	(1.283)	(2.734)	(0.655)	(1.409)	(2.887)
Observations	551	521	521	495	468	468
Number of id			10			9

PDR - provincail debt per revenue, CF - control function correction term
Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4: Probability of Internal Conflicts

3. Theoretical framework

Our theoretical framework depicts a federal union including N districts, indexed by $s \in \{1, \cdots, N\}$, each populated by $M > 1$ citizens. In each district, there is an incumbent (denoted by I) who seeks re-election and a challenger (denoted by O). We focus on a particular district, that is, district i. Building on a standard probabilistic voting setup, we model a two-period game (before and after the election) between the incumbent and voters.

3.1. The multi-district setup

In each period $t \in \{1, 2\}$, the incumbent provides a public good ($g_{t,i}$) and levies taxes from output ($\tau Y_{t,i}$, with τ the constant tax rate and $Y_{t,i}$ the level of output).

In the first period, the deficit must be financed by borrowing from the federal union $\mathcal{D}_i = g_{1,i} - \tau Y_{1,i}$. At the end of the first period, an election takes place in district i and the incumbent is re-elected or his/her challenger takes office.

In the second (and last) period, the newly elected politician has to repay public debt and interest since he/she cannot borrow in the last period ($\mathcal{D}_{2,i} = 0$). However, the sharing of the debt burden is subject to conflicts within the federation since each district attempts to avoid reimbursement.[15] In this stage, the district i will have to repay a part ψ_i of the debt burden $(1 + r)\mathcal{D}_i$ ($r \geq 0$ is the constant interest rate). This part will be endogenously determined in the following sections. Thus, the government's budget constraint is, in period 2

$$\mathcal{D}_{2,i} = g_{2,i} + \psi_i(1 + r)\mathcal{D}_i - \tau Y_{2,i} = 0. \tag{5}$$

If $\psi_i < 1$, district i's citizens benefit from debt alleviation (or partial default), while they must pay for their neighbors in the opposite case ($\psi_i > 1$). In the equilibrium, however, all the public debt issued by the federation has to be repaid; hence, $\sum_{s=1}^{N} \psi_s \mathcal{D}_s = \sum_{s=1}^{N} \mathcal{D}_s$, and the symmetric equilibrium will require $\psi_s = 1, \forall s$.

The intertemporal utility of citizen n, $n \in \{1, \cdots, M\}$, is assumed to be linear (as we focus on a particular district, we drop the subscript i hereafter, except when necessary):[16]

$$U(g_{1,n}, g_{2,n}) = g_{1,n} + \beta g_{2,n} =: U_n, \tag{6}$$

where $\beta \in (0, 1)$ is the discount factor and $g_{t,n}$ denotes the amount of public good received by citizen n belonging to district i in period t, with $g_t = \sum_{n=1}^{M} g_{t,n}$.

Owing to imperfect property rights, the public good is rival but non-excludable, and

[15] After the election, the debt burden can be renegotiated inside the federation. Some districts can obtain debt alleviations, bailouts, or specific assistance from the federal budget depending on their bargaining power. Since the federal budget must be balanced, other districts will have to pay for these measures, which produces inter-district conflicts about the repayment of public debt.

[16] We could also assume that there is a private good and that citizens' utility (6) is consumption-augmented. In this case, citizens consume their net income ($(1 - \tau)Y_t/M$ in a symmetric configuration), and our results are qualitatively unchanged.

the amount $g_{t,n}$ that citizen n can acquire is subject to conflicts.[17] Thus, citizens are induced to provide rent-seeking efforts to capture the largest share of public good g_t (yet to be derived) that defines the rent available in district i. The rent in the two periods is

$$g_1 = \tau Y_1 + \mathcal{D}, \tag{7}$$

$$g_2 = \tau Y_2 - \psi(1+r)\mathcal{D}. \tag{8}$$

Additionally, as discussed above, the property rights on the federal debt burden are imperfectly defined. Thus, by engaging in collective post-electoral fighting against other districts, district i's citizens can expect to reduce the debt burden they will face, thereby producing an extra incentive to carry out rent-seeking activities.

Consequently, two kinds of conflicts can be distinguished: *internal conflicts* (among the citizens of district i), and *external conflicts* (between the citizens of district i and those of other districts), which are explained in the following subsections.

3.2. Internal conflicts

In district i, citizen n provides rent-seeking effort $x_{t,n}$ to acquire a part of public good g_t. Let $j \in \{I, O\}$ denote the politician who holds power ($j = I$ in the first period, and $j = I$ or O in the second period, if the incumbent is re-elected or not, respectively), and α^j define the share of the captured public good, namely

$$g_{t,n} = \alpha^j(\mathbf{x}_t) g_t. \tag{9}$$

where $\mathbf{x}_t := (x_{t,1}, \cdots, x_{t,M})$ is the vector of citizens' effort. As usual, we assume a logistic contest success function:[18]

$$\alpha^j(\mathbf{x}_t) = \frac{(x_{t,n})^{a^j}}{\sum_{m=1}^{M} (x_{t,m})^{a^j}}, \tag{10}$$

where $a^j \geq 0$ is the *decisiveness* of the internal conflict (see, e.g., Hirshleifer, 1989). Indeed, the higher a^j, the higher is the expected efficiency of the rent-seeking effort. Following Hirshleifer (2008) and Münster and Staal (2011), a^j is determined by both the institutional and political factors affecting the protection of property rights. Consequently, the decisiveness of the internal contest is likely to depend on the office-holder

[17] Strictly speaking, this public good is an impure public good (i.e., a common good).
[18] An axiomatic foundation is provided by Skaperdas (1996).

(j). One way of explaining this feature is that politicians have different abilities to secure property rights inside their jurisdiction (Hirshleifer, 2008, p.21). In our model, while politicians have no intrinsic preferences or competencies, the simple fact that the incumbent is re-elected or the challenger is acceded to power can change the incentives of citizens to fight. This change in voters' incentives will affect the decisiveness parameter, which becomes dependent on the "type" of politician $j = I, O$.[19] For example, if the challenger is elected, he/she may benefit from a "honeymoon effect", since citizens are less induced to fight if they face a virgin politician (see, e.g., Mueller, 1973; Kernell, 1978). Following the incumbent's defeat, the in-place power networks are destroyed, the office staff is re-elected, and old barriers to entry fall in such a way that the decisiveness of the internal conflict (a^j) decreases; hence, $a^O \leq a^I$. For the sake of completeness, we consider in the following any positive or negative value of $a^O - a^I$.

3.3. External conflicts

In the second period, citizen n can provide an additional fighting effort $y_{2,i,n}$ to avoid debt repayment. The mechanism underlying external conflicts differs from internal rent-seeking because the expected gain from such conflicts depends on the collective action of district i's citizens and government, and this involves some coordination inside the district. Indeed, the government can be induced to organize the collective external fighting effort (e.g., by allowing or instigating growing public protests against the burden of the federal debt) to consolidate its popularity. As we show below, coordinating the struggle against a common foreign enemy is a means for an opportunistic government to divert citizens from internal social conflicts to its benefit.[20]

Let us define the collective effort of district i by $\hat{y}_{2,i} := e_i \sum_{n=1}^{M} y_{2,i,n}$, where e_i is the government's coordination effort. After the election, citizens of district i must repay a debt burden $\psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}$, where $\hat{\mathbf{y}}_2 := (\hat{y}_{2,1}, \cdots, \hat{y}_{2,N})$. Similar to internal conflicts, we assume a logistic contest success function, namely

$$\psi(\hat{\mathbf{y}}_2) = \frac{\hat{y}_{2,i}^{-b}}{\sum_{s=1}^{N} \hat{y}_{2,s}^{-b}}, \tag{11}$$

[19] Thus, in the second period, the mere fact of having been the incumbent or challenger in the election may roughly characterize the "type" of office-holder.

[20] This finding echoes the traditional diversionary hypothesis (see Wright, 1965; Mansfield and Snyder, 2004), which suggests that leaders might trigger conflict with other groups to deflect attention from problems at home. Wright states that *"foreign war as a remedy for internal tension, revolution, or insurrection has been an accepted principle of government"* (Wright, 1965, p.140).

where $b \geq 0$ describes the *decisiveness* of the external contest. Note that b differs from a^j since the contest between districts within the federal union is subject to different rules than that between individuals within a particular district.

In the symmetric equilibrium, external conflicts will be unproductive, but detrimental to output and welfare (like internal conflicts) because each district will repay the same amount $(1+r)\sum_{s=1}^{N} \mathcal{D}_s/N$ of debt (i.e., $\psi_s = 1$ for any s). This inefficiency is larger given the higher decisiveness parameter b.[21]

3.4. Citizens' utility

Taking into account rent-seeking activities, citizen n's utility (6) becomes

$$U_n^j = \alpha^I(\mathbf{x}_1)\left[\tau Y_1 + \mathcal{D}\right] + \beta\alpha^j(\mathbf{x}_2)\left[\tau Y_2 - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right]. \tag{12}$$

Beyond engaging in conflict, citizens provide productive efforts $h_{t,n}$ that yield output $Y_t := \sum_{n=1}^{M} h_{t,n}$. Thus, their budget constraint is, in the first period[22]

$$x_{1,n} + h_{1,n} = 1,$$

and, in the second period

$$x_{2,n} + y_{2,n} + h_{2,n} = 1.$$

Hence,

$$Y_1 = \sum_{n=1}^{M}(1 - x_{1,n}), \text{ and } Y_2 = \sum_{n=1}^{M}(1 - x_{2,n} - y_{2,n}). \tag{13}$$

Therefore, by diverting effort from productive destinations, rent-seeking activities are costly. From (12) and (13), if politician j is elected, citizen n's utility is

$$U_n^j(x_{1,n}, x_{2,n}, y_{2,n}) = \alpha^I(\mathbf{x}_1)\left[\tau \sum_{n=1}^{M}(1 - x_{1,n}) + \mathcal{D}\right] + \beta\alpha^j(\mathbf{x}_2)\left[\tau \sum_{n=1}^{M}(1 - x_{2,n} - y_{2,n}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right], \tag{14}$$

[21] To simplify our analysis, the decisiveness parameter of external conflicts (b) does not depend on the type of office-holder, contrary to that of internal conflicts (a^j). However, we could easily introduce different parameters such that $b^I \neq b^O$. This assumption would unnecessarily complicate the model without affecting our results.

[22] Each citizen is endowed with one unit of time. For simplicity, citizens simultaneously and independently decide how to allocate their resources to productive and unproductive activities.

under the non-negativity constraint $g_2 \geq 0$.

From (14), the level of rent in the second period has a negative association with the efficiency of the external fighting effort (ψ). This will only reduce the incentive to fight internally (through the term α^j). In the equilibrium, this will produce a negative relationship between both types of conflicts. This result is presented below.

4. Political Competition

Let us now describe the electoral side of the model. Citizens have preferences for ideology and other politicians' characteristics. Thus, in district i, citizen n receives additional expected utility $(\theta_n + \xi)$ if politician I takes power. To avoid generating a deterministic election outcome, this term includes two random components: θ_n is idiosyncratic and ξ is common to all voters. Following the probabilistic voting models of Lindbeck and Weibull (1987) and Persson and Tabellini (2000), θ_n are independent random variables, constant over time, and uniformly distributed on $[-1/2w, 1/2w]$, with density $w > 0$,[23] and ξ reflects the (relative) general popularity of politician I, which is uniformly distributed on $[1/2h, 1/2h]$, with density $h > 0$. Then, if politician j is elected, citizen n's expected utility becomes

$$\mathbb{E}U_n^j(x_{1,n}, x_{2,n}, y_{2,n}) := \begin{cases} U_{i,n}^I(x_{1,n}, x_{2,n}, y_{2,n}) + \theta_n + \xi & \text{if } j = I, \\ U_{i,n}^O(x_{1,n}, x_{2,n}, y_{2,n}) & \text{if } j = O. \end{cases} \tag{15}$$

4.1. Politician's objective

The exercise of power generates psychological gains and losses for the office-holder. First, an (exogenous) ego-rent $R > 0$ is perceived in each period. Second, after the election, there are additional (endogenous) net gains because the elected politician benefits from extra ego-rents, in the form of "popularity" gains, if the district succeeds in reducing the debt burden. His/her second-period rent then increases with the gain $1 - \psi(\cdot)$. These extra rents motivate the office-holder to engage in a costly effort (e) to avoid repaying the debt.[24] Hence, the total psychological rents R_t that the office-holder perceives are, in

[23] A positive (resp. negative) value of θ_n implies that citizen n has a bias in favor of the incumbent (resp. the challenger), whereas citizens with $\theta_n = 0$ are ideologically neutral. Further, w measures citizens' responsiveness to rent-seeking activities. As w increases, citizens care more about rent captation than ideology (see Eq. 15).

[24] Many empirical works support this specification. For example, the well-known *"rally 'round the flag syndrome"* suggests that external conflicts often boost the popularity of leaders (Grieco et al., 2014,

the first period[25]

$$R_1 = R,$$

and in the second period, assuming a linear cost of effort,

$$R_2 = R + \lambda(1 - \psi(\hat{\mathbf{y}}_2))(1 + r)\mathcal{D} - e, \text{ with } \frac{\partial \hat{\mathbf{y}}_2}{\partial e} \geq 0.$$

The parameter $\lambda \geq 0$ reflects how citizens reward the office-holder when the district succeeds in external fighting ($\psi < 1$). In the opposite case ($\psi > 1$), the government suffers from a popularity loss. A politician in search of popularity will then undertake efforts to coordinate the external fight, even if, in the symmetric equilibrium, this fight will be unproductive ($\psi = 1$).

The incumbent's objective is to maximize the intertemporal flow of expected power rents, namely

$$\mathbb{E}[V] = R_1 + \beta\mu R_2, \tag{16}$$

where $\mathbb{E}$ denotes the expectation operator (with expectations taken over the election outcome) and μ is the (endogenous) reelection probability.

The office-holder has a unique strategic variable in each period. In the first period, the incumbent sets the amount of public debt $\mathcal{D}$. In the second period, the elected politician determines his/her coordination effort e, conditional on the amount of debt initially chosen. Hence, when the incumbent decides $\mathcal{D}$, he/she must take into account the effect of this action on his/her optimal coordination effort if re-elected. The following subsection details the timing of the model.

4.2. Timing of the game

The timing of events is as follows:

1. **Period 1.** The incumbent chooses the amount of debt $\mathcal{D}$. In this stage, all agents know the distributions of θ_n and ξ, but not their realized values.
2. Citizen n provides a rent-seeking effort $x_{1,n}$; the actual value of ξ is realized, all uncertainty is resolved, and election is held.

p.199). For example, some US presidents have enjoyed extra short-run popular support during the outbreak of international crises or wars (as, e.g., Lian and Oneal, 1993; Hetherington and Nelson, 2003, among others).

[25] Power rents are assumed to be purely psychological. This avoids introducing an unnecessary additional conflict between the government and district i citizens.

3. **Period 2.** The incumbent is re-elected ($j = I$) or the challenger is elected ($j = O$).
4. The newly elected politician implements coordination effort e. Citizen n makes rent-seeking efforts $x_{2,n}$ and $y_{2,n}$, and the game ends.

As usual, we look for the Subgame Perfect Equilibrium and we solve the model by backward induction. The two stages (after and before the election) are respectively depicted in the following sections.

5. Second-period equilibrium

In the second period, the office-holder calculates the optimal coordination effort and citizens choose their optimal fighting efforts conditional on the public debt.

5.1. The government's optimal coordination effort

The newly elected politician $j \in \{I, O\}$ chooses the coordination effort e that maximizes his/her second-period power rent:

$$R_2 = R + \lambda(1 - \psi(\hat{\mathbf{y}}_2))(1 + r)\mathcal{D} - e. \tag{17}$$

Hence, the following first-order condition is

$$-\lambda(1 + r)\mathcal{D}\frac{\partial\psi(\hat{\mathbf{y}}_2)}{\partial e} = 1. \tag{18}$$

The optimal effort is such that the marginal cost simply equals the marginal gain. By producing one additional unit of effort, the office-holder benefits from the extra psychological gain from the marginal increase in the collective surplus (the LHS of Eq. (18), which is positive because $\partial\psi(\cdot)/\partial e \leq 0$), but incurs one unit of marginal cost through the simple linear cost function.

Focusing on the symmetric equilibrium, which we describe below, we have $\hat{y}_{2,s} =: \hat{y}_2$, for any s; moreover, from Eq. (11), the optimal coordination effort is[26]

$$e^*(\mathcal{D}) = \lambda\tilde{N}b(1 + r)\mathcal{D}, \tag{19}$$

[26] Indeed, in the symmetric equilibrium, we have $\partial\psi(\hat{\mathbf{y}}_2)/\partial e = -b\tilde{N}/e$. The second-order condition is verified since $e \mapsto \psi(\hat{\mathbf{y}}_2)$ is a convex mapping.

where $\tilde{N} := (N-1)/N^2 > 0$.

The optimal effort positively depends on public debt $\mathcal{D}$. This dependence is sensitive to the decisiveness parameter (b). Indeed, the higher public debt, the higher are the potential gain from external fighting and the marginal gain from the coordination efforts. Further, as decisiveness increases, district i is more likely to avoid debt repayment, which increases the government's incentive to coordinate the conflict.

From (17), the second-period power rent is, in the symmetric equilibrium ($\psi = 1$),

$$R_2^* = R - e^*(\mathcal{D}) = R - \lambda \tilde{N} b(1+r)\mathcal{D} =: R_2^*(\mathcal{D}).$$

In the equilibrium, the office-holder's payoff negatively depends on public debt, given his/her unproductive coordination efforts to avoid repaying the debt burden.

5.2. Equilibrium internal and external rent-seeking activities

Given the government's effort e^*, citizens optimally determine their rent-seeking activities $x_{2,n}$ and $y_{2,n}$. If politician j holds office, citizen n's programme is, by (6),

$$\begin{aligned} &\max_{(x_{2,n}, y_{2,n}) \in \mathscr{C}} \mathbb{E}U_n^j(x_{1,n}, x_{2,n}, y_{2,n}), \\ &\text{s.t. } \tau \sum_{s=1}^{M} (1 - x_{2,s} - y_{2,s}) \geq \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}, \end{aligned} \tag{20}$$

where $\mathscr{C} := \{(x, y) \in [0, 1]^2;\ x + y \leq 1\}$.

Using (14), the first-order condition on $x_{2,n}$ is (the strict complementarity slackness condition holds at the equilibrium, see Appendix B)

$$\frac{\partial \alpha^j(\mathbf{x}_2)}{\partial x_{2,n}} \left[\tau \sum_{s=1}^{M} (1 - x_{2,s} - y_{2,s}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D} \right] = \alpha^j(\mathbf{x}_2)\tau. \tag{21}$$

The LHS of (21) is the marginal gain from acquiring an additional share of the public good by exerting one more unit of intra-district effort ($g_2 \partial \alpha^j / \partial x_{2,n}$). This marginal gain negatively depends on $\mathcal{D}$, because the debt burden reduces the amount of the feasible public good (g_2). The RHS is the marginal cost, which is the opportunity cost of rent-seeking activities in terms of fiscal resources ($\alpha^j \tau$).

Using (14), the first order condition on $y_{2,n}$ is

$$\tau = -\frac{\partial \psi(\hat{\mathbf{y}}_2)}{\partial y_{2,n}}(1+r)\mathcal{D}. \tag{22}$$

The LHS of (22) is the marginal cost of external rent-seeking behavior, which, as previously, corresponds to the loss of fiscal resources (τ). The RHS represents the marginal gain from fighting. This gain positively depends on $\mathcal{D}$, which is the stake of external conflict.

In our model, citizens differ only by their intrinsic ideological preference θ_n. As they share the same utility function and face similar budget constraints, we use the concept of symmetric equilibrium, namely citizens simultaneously and independently solve the same programme (20), and the optimal rent-seeking efforts are characterized by $x_{t,n} =: x_t$ and $y_{2,n} =: y_2$. The first-order conditions (21) and (22) then lead to (see Appendix B)

$$\frac{a^j}{x_2}\left(\frac{M-1}{M}\right) = \frac{\tau}{\tau M(1-x_2-y_2)-(1+r)\mathcal{D}}, \tag{23}$$

$$\tau = \frac{b\tilde{N}(1+r)}{y_2 M}\mathcal{D}. \tag{24}$$

The following proposition establishes the unique couple of efforts at the symmetric equilibrium.

Proposition 1. *Let $\kappa := (1+r)/\tau M$. The unique optimal set of rent-seeking efforts $(x_2^{*j}, y_2^*) \in \mathscr{C}$ is, for any $\mathcal{D} \in [0, \bar{\mathcal{D}}]$,*

$$x_2^{*j} = A^j\left[1-\kappa B\mathcal{D}\right] =: x_2^{*j}(\mathcal{D}), \tag{25}$$
$$y_2^* = \kappa(B-1)\mathcal{D} =: y_2^*(\mathcal{D}), \tag{26}$$

where $A^j := a^j(M-1)/(1+a^j(M-1)) < 1$, $B := 1+b\tilde{N}$, and $\bar{\mathcal{D}} := \{\mathcal{D}|x_2^{*j} = 0; j = I, O\}$ corresponds to the highest public debt level consistent with positive effort $x_2^{*j} \geq 0$.

Proof: See Appendix B.

The optimal effort in internal conflict (x_2^{*j}) depends on the type of politician (j) in office, through the term A^j (which is equivalent to a^j). Indeed, from (25), internal rent-seeking activities are positively related to the decisiveness parameter (i.e., $\partial x_2^{*j}/\partial a^j > 0$), which increases the expected efficiency of the effort and the marginal gain from fighting.

In addition, x_2^{*j} negatively depends on b. Effectively, the higher b, the sharper is the external conflict. This reduces production and the available public good, which is the basis of the internal conflict. Hence, there is less inducement to fight inside the district. There is no corresponding inverse relation between a^j and y_2^* because the basis of the external conflict (the public debt burden) does not depend on the severity of the internal conflict.[27]

Proposition 1 highlights one outstanding feature. By issuing debt in the first period, the incumbent can generate a tradeoff between both types of conflicts. Indeed, public debt reduces the basis of the intra-district conflict by reducing the feasible public good, while broadening the basis of the external conflict through the fight against repaying the debt.

Figure 1 describes this tradeoff. Since $(x_2^{*j})'(\mathcal{D}) < 0$ and $(y_2^*)'(\mathcal{D}) > 0$, as public debt increases, the equilibrium moves from point E_1 to point E_3 along the parametric curve with respect to $\mathcal{D}$.

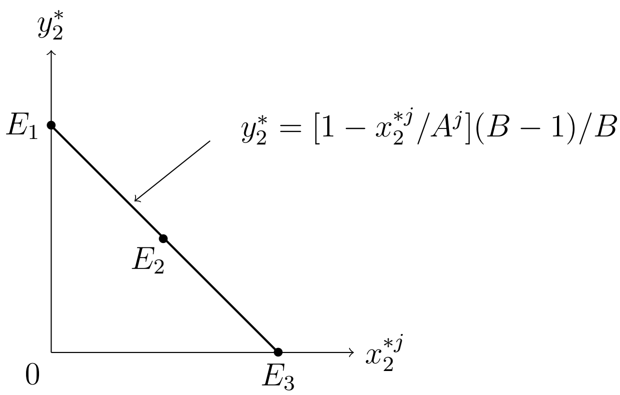

Figure 1: The tradeoff between internal and external conflicts

This tradeoff is closely related to the Sumner (1906)'s well-known concept of ethnocentrism, whereby *"the relation of comradeship and peace in the we-group and that of hostility and war towards the other-groups are correlative to each other"* (Sumner, 1906, p. 12). Our model offers a novel perspective of this mechanism, based on public debt management. Similar to the diversionary theory of war, public debt can be used to create diversion in managing internal and external social conflicts. Through public debt, the incumbent can generate, after the election, internal conflict only (point E_1 in Figure 1),

[27]Hence, following Münster (2007)'s definition, our model exhibits only a "reversed group cohesion" effect.

external conflict only (point E_3), or any combination of internal and external conflicts (as in point E_2).

By directing citizens' fighting behavior toward the rest of the federation rather than themselves, public debt is then a political instrument in the hands of the incumbent. Therefore, a strategic incumbent can manage public debt to secure social peace during his/her possible future mandate for his/her personal interest.

6. First-period equilibrium

This section solves the first-period equilibrium by two-step backward induction. In the first step, citizens compute their rent-seeking effort and, in the second step, the incumbent determines the amount of debt.

6.1. First-period rent-seeking behavior

Given the optimal levels of the second-period effort (x_2^{*j}, y_2^*), citizen n optimally determines his/her first-period effort $x_{1,n}$. In the symmetric equilibrium, the optimal effort x_1^{*I} satisfies

$$x_1^{*I} := \underset{x_1 \in [0,1]}{\operatorname{argmax}} \, \mathbb{E} U_n^j(x_1, x_2^{*j}, y_2^*).$$

From (14), the first-order condition is

$$\frac{\partial \alpha^I(\mathbf{x}_1)}{\partial x_{1,n}} \left[\tau \sum_{s=1}^{M} (1 - x_{1,s}) + \mathcal{D} \right] = \alpha^I(\mathbf{x}_1)\tau. \tag{27}$$

As in the previous section, the LHS of (27) represents the marginal gain from obtaining additional rents, while the RHS is the marginal opportunity cost of rent-seeking activities. In the symmetric equilibrium ($x_{1,n} =: x_1$), Eq. (27) becomes $a^I(M-1)/M^2 x_1 = \tau/[\tau M(1 - x_1) + \mathcal{D}]$, and the optimal effort positively depends on public debt, which increases the available public good in the first period, namely[28]

$$x_1^{*I} := A^I \left[1 + \frac{\kappa \mathcal{D}}{1 + r} \right] =: x_1^{*I}(\mathcal{D}). \tag{28}$$

[28]The second-order condition is satisfied since $(\alpha^I)''(\cdot) < 0$ (see Appendix B).

6.2. The voting process

At the end of period 1, the election takes place and citizens vote for the candidate who brings them the highest expected utility. Citizen n supports the incumbent (politician I) if and only if

$$\mathbb{E}U_n^{*I}(\mathcal{D}) > \mathbb{E}U_n^{*O}(\mathcal{D}),$$

where $\mathbb{E}U_n^{*j}(\mathcal{D}) := \mathbb{E}U_n^j(x_1^{*I}(\mathcal{D}), x_2^{*j}(\mathcal{D}), y_2^*(\mathcal{D}))$, namely, from (15), iff

$$\theta_n > \overline{\theta} := -\beta\Delta(\mathcal{D}) - \xi,$$

with $\Delta(\cdot) := U_n^{*I}(\cdot) - U_n^{*O}(\cdot)$ the differential of welfare.

Citizens with $\theta_n > \overline{\theta}$ prefer politician I. Thus, given our assumptions about the distribution of ideological preferences, politician I's vote share is

$$\pi = \sum_{n=1}^{M} \mathbb{P}\{\theta_n > \overline{\theta}\} = \sum_{n=1}^{M} \int_{\overline{\theta}}^{1/2w} w \, dz = \frac{M}{2} - Mw\overline{\theta}. \tag{29}$$

From both candidates' points of view, π is a random variable since it is a transformation of the random shock ξ. The electoral outcome is thus a random event, related to the realization of the shock ξ. Let us consider a majoritarian rule setting in which the politician obtaining more than 50% of the votes wins the election. Under this rule, the reelection probability of politician I is

$$\mu = \mathbb{P}\left\{\pi \geq \frac{M}{2}\right\} = \mathbb{P}\left\{\xi \geq -\Delta(\mathcal{D})\right\}. \tag{30}$$

Hence, given our assumptions about the distribution of ξ,[29]

$$\mu = \frac{1}{2} + h\Delta(\mathcal{D}) =: \mu(\mathcal{D}). \tag{31}$$

Using (14), as the optimal effort expended on external fighting (y_2^*) does not depend on the type of politician in power, the differential of welfare is, in the symmetric equilibrium ($\alpha^j = 1/M$)

$$\Delta(\mathcal{D}) = \beta\tau[x_2^{*O}(\mathcal{D}) - x_2^{*I}(\mathcal{D})].$$

[29] There is an interior solution, provided that h is sufficiently small.

For tractability convenience, let us define the differentials of decisiveness parameter $\tilde{A} := (A^I - A^O)/2$ and the average $\bar{A} := (A^I + A^O)/2$. From Proposition 1, it follows that $\Delta(\mathcal{D}) = 2\tau\beta(\kappa\mathcal{D}B\tilde{A} - \tilde{A})$, hence

$$\mu(\mathcal{D}) = \frac{1}{2} + 2h\tau\beta\left(\kappa\mathcal{D}B\tilde{A} - \tilde{A}\right). \tag{32}$$

The re-election probability ($\mu(\mathcal{D})$) depends on public debt through the comparative advantages or disadvantages of politicians in managing the internal conflict. The term $\tilde{A}$ reflects citizens' comparative incitement to fight if the incumbent rather than the challenger is elected. Any increase in $\tilde{A}$ exerts two conflicting effects. First, the higher $\tilde{A}$, the higher is the internal conflict if the incumbent is re-elected (this corresponds to the negative effect in Eq. 32). Second, there is a positive effect in (32) due to the interaction between both types of conflicts: more internal conflict leads to less external conflict, as already emphasized.

In this way, if $\tilde{A} > 0$, $\mu(\mathcal{D})$ positively depends on $\mathcal{D}$. Effectively, citizens know that internal conflict will rise if the incumbent is elected rather than his/her challenger. Through this channel, public debt increases the re-election probability by reducing internal conflicts and the comparative disadvantage of the incumbent.

However, the incumbent's objective is not to maximize the probability of re-election, but the expected value of intertemporal power rents. We now turn our attention to the determination of the optimal level of public debt.

7. Public debt and social cohesion

In the first period, the incumbent sets the public debt $\mathcal{D}$ that maximizes his/her expected intertemporal payoff. He/she internalizes the consequences of his/her choice on citizens' rent-seeking activities and on his/her coordination effort in the second period if re-elected. Thus, using (16), the incumbent maximizes

$$\mathbb{E}[V] = R_1 + \beta\mu(\mathcal{D})R_2^*(\mathcal{D}) = R + \beta\mu(\mathcal{D})[R - e^*(\mathcal{D})].$$

The solution to this problem leads to the following first-order condition:

$$\mu'(\mathcal{D})[R - e^*(\mathcal{D})] = \mu(\mathcal{D})(e^*)'(\mathcal{D}). \tag{33}$$

The incumbent chooses public debt such that the marginal gain from being re-elected equals the marginal effort cost if re-elected. The LHS of (33) is the marginal gain from issuing public debt, namely the marginal increase in the probability of being elected ($\mu'(\mathcal{D})$), adjusted by the power rent ($R - e^*(\mathcal{D})$). The RHS of (33) is the marginal cost of debt: high public debt today forces the incumbent (if re-elected, with probability μ) to undertake high effort tomorrow.

As discussed above, the reelection probability positively ($\mu' > 0$) or negatively ($\mu' < 0$) depends on public debt according to the sign of $\tilde{A}$. As $(e^*)' > 0$ and $\mu > 0$, we obtain an interior maximum only if $\tilde{A} > 0$. In the opposite case, there is a corner solution, as stated in the following proposition.

Proposition 2. *(Equilibrium Characterization) There are two positive critical values $\tilde{A}_1$ and $\tilde{A}_2$ (where $0 < \tilde{A}_1 < \tilde{A}_2$), such that the unique equilibrium $\mathcal{D}^*$ is characterized by the three following cases.*

i. *If $\tilde{A} \leq \tilde{A}_1$, $\mathcal{D}^* = 0$.*

ii. *If $\tilde{A} \geq \tilde{A}_2$, $\mathcal{D}^* = \bar{\mathcal{D}}$.*

iii. *If $\tilde{A}_1 < \tilde{A} < \tilde{A}_2$,*

$$\mathcal{D}^* = \bar{R} + \frac{\tilde{A} - \Psi}{2\kappa B \tilde{A}},$$

where $\Psi := (4h\beta\tau)^{-1}$ and $\bar{R} := R[2\lambda(1+r)(B-1)]^{-1}$.

Proof: See Appendix C.

Figure 3 depicts the optimal debt strategy of the incumbent and the corresponding equilibrium level of internal and external conflicts in period 2 if he/she is re-elected in function of the decisiveness gap $\tilde{A}$.

If $\tilde{A} > 0$, the incumbent has a comparative disadvantage in internal conflicts relative to his/her challenger. As $\tilde{A}$ increases, it is in the incumbent's interest to raise public debt to reduce this disadvantage. Indeed, by cutting the stake of the internal conflict, public debt allows reducing internal rent-seeking activities during the second period (x_2^{*I} decreases). For $\tilde{A}_1 < \tilde{A} < \tilde{A}_2$, the optimal debt strategy gives rise to an interior solution $0 < \mathcal{D}^* < \bar{\mathcal{D}}$. Noteworthily, without any comparative advantage or disadvantage ($\tilde{A} = 0$), the incumbent never issues public debt ($\mathcal{D}^* = 0$). Indeed, the office-holder is induced to issue public debt only if his/her comparative advantages are sufficiently high to offset the coordination effort he/she will have to undertake, if re-elected.

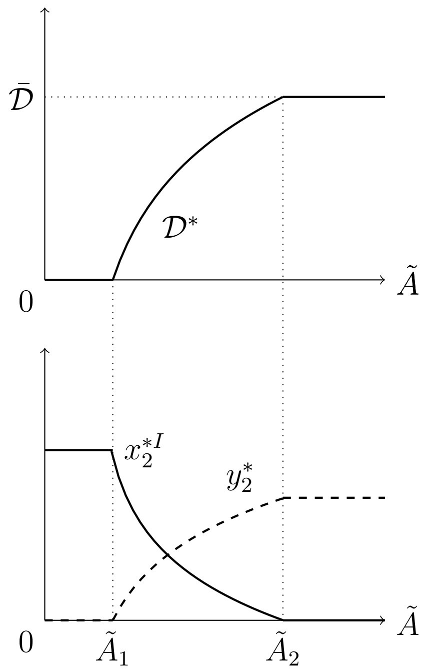

Figure 3: Optimal debt and conflicts in function of the decisiveness gap

Figure 3 highlights the strategic use of public debt as a diversionary device. This diversionary tactic is maximal if the comparative advantage of the opponent in internal conflict resolution is high, i.e. if $\tilde{A} > \tilde{A}_2$. By contrast, the incentive to use public debt for diversionary motives disappears if $\tilde{A} < \tilde{A}_1$.

Finally, to establish a benchmark, the following subsection determines the optimal public debt that maximizes social welfare in district i.

8. Social welfare

The intertemporal social welfare of district i (W) is the sum of all citizens' payoffs, and this depends on the type of politician who takes office in the second period. Therefore, expected social welfare is

$$\mathbb{E}[W] = \sum_{n=1}^{M} W_{1,n}^{I} + \beta\mu \sum_{n=1}^{M} W_{2,n}^{I} + \beta(1-\mu) \sum_{n=1}^{M} W_{2,n}^{O}, \tag{34}$$

where $W_{t,n}$ is the payoff of citizen n. Using the intertemporal utility (14), we have

$$W_{1,n}^I = \alpha^I(\mathbf{x}_1)\left[\tau\sum_{n=1}^{M}(1-x_{1,n})+\mathcal{D}\right],$$
$$W_{2,n}^j = \alpha^j(\mathbf{x}_2)\left[\tau\sum_{n=1}^{M}(1-x_{2,n}-y_{2,n})-\psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right].$$

In the symmetric equilibrium ($W_{t,n} =: W_t$, for any n), expected social welfare (34) becomes

$$\mathbb{E}[W] = MW_1^I + \beta M\mu[W_2^I - W_2^O] + \beta MW_2^O, \tag{35}$$

thereby, using optimal effort levels (25), (26) and (28)

$$\mathbb{E}[W](\mathcal{D}) = M\Delta(\mathcal{D})\left[\frac{1}{2}+h\Delta(\mathcal{D})\right]+\beta\tau M(1-x_2^{*O}-y_2^*)+[1-\beta(1+r)]\mathcal{D}+\tau M(1-x_1^{*I}). \tag{36}$$

The following proposition determines the socially optimal level of public debt ($\mathcal{D}^{SW}$).

Proposition 3. *Let $\beta = 1/(1+r)$. We have:*
i. If $\tilde{A} \geq 0$, $\mathcal{D}^{SW} = 0$.
ii. If $\tilde{A} < 0$, there is a critical level $\bar{B} > 1$, such that $\mathcal{D}^{SW} = \bar{\mathcal{D}}$ if $B < \bar{B}$, and $\mathcal{D}^{SW} = 0$ if $B \geq \bar{B}$.

Proof: By inspecting Eq. (36), note that $\mathcal{D} \mapsto \mathbb{E}[W]$ is a quadratic function, namely $\mathbb{E}[W] = \epsilon_1\mathcal{D}^2 + \epsilon_2\mathcal{D} + \epsilon_3$, where $\epsilon_1 := Mh[2\tau\kappa B\tilde{A}]^2 > 0$. Thus, $\mathcal{D} \mapsto \mathbb{E}[W]$ describes a U-shaped curve and the social planner of district i maximizes the expected social welfare (36) by issuing public debt $\mathcal{D}^{SW} \in \{0, \bar{\mathcal{D}}\}$. Appendix D characterizes the socially optimal level of public debt in function of the decisiveness parameter. □

As welfare is a U-shaped function of public debt in Eq. (36), no interior solution can appear and the optimum is a bang-bang equilibrium: the social planner chooses either zero or maximal ($\bar{\mathcal{D}}$) public debt. Indeed, the social planner internalizes the second-period citizens' payoffs for both election outcomes (i.e. W_2^I and W_2^O). Specifically, in Eq. (36), expected welfare is quadratic in the gap of payoffs $\Delta = W_2^I - W_2^O$, and thus society enjoys extreme values of Δ.

Effectively, it is important, for citizens' welfare, that the politician who brings about the highest utility is likely to be elected. This is the case if Δ is strongly positive (the incumbent brings about higher gains than the challenger and has a high probability of re-election) or strongly negative (the challenger brings about higher gains than the incumbent and is more likely to be elected). Proposition 3 shows that such extreme values of Δ arise if $\mathcal{D} = \bar{\mathcal{D}}$ or $\mathcal{D} = 0$, depending on the gaps of decisiveness in conflicts.

Interestingly, if the challenger has a comparative advantage in internal conflicts ($\tilde{A} > 0$), the equilibrium is suboptimal, with an overaccumulation of public debt ($\mathcal{D}^* > \mathcal{D}^{SW} = 0$). However, if the incumbent has a comparative advantage ($\tilde{A} < 0$), the equilibrium can be optimal, with $\mathcal{D}^* = \mathcal{D}^{SW} = 0$, provided that $B \geq \bar{B}$. Consequently, having an equilibrium characterized by positive public debt cannot maximize social welfare.

9. Conclusion

Social science research agrees on the positive relationship between intergroup conflicts and group cohesion (see Tajfel, 1982, for a review). In a relevant paper, Stein (1976) concludes that "*there is a clear convergence in the literature in both the specific studies and in the various disciplines that suggests external conflict does increase internal cohesion*" (Stein, 1976, p.165).

In this study, we developed such an idea in the context of a macroeconomic model in which a strategic incumbent manages public debt to monitor internal and external conflicts in function of his/her comparative advantage relative to the challenger. Specifically, by increasing public debt, an incumbent whose challenger benefits from a comparative advantage in managing internal conflicts can lessen this advantage by reducing expected rent-seeking behavior if re-elected. By doing this, he/she ties his/her hands to help avoid citizens' claims during his/her term.

Additionally, by developing external conflicts, public debt can be even more profitable for the incumbent, who will benefit from fewer external conflicts than his/her challenger if re-elected. This double dividend of public debt remains even if, in the symmetric equilibrium, the incumbent does not obtain any popularity advantage from the conflict with other districts.

Our study is built on the traditional idea that competition over public expenditure, or over any resource whose property rights are imperfectly defined, gives rise to social conflicts or rent-seeking activities. A number of examples show that windfall budget surpluses exacerbate claims from various social groups (as highlighted by the famous episode of the "fiscal jackpot" provided by the resumption of French economic growth in 2000).[30] Other prominent examples emphasize the role of public debt in channeling citizens' anger against international institutions or foreign powers. For instance, in the current decade, hampered by the burden of a huge debt, the Greek government recently encouraged protests against foreign creditors (the well-known *"Troïka"*).[31] The resulting high public debt then served as a political instrument both to continue international negotiations and to force citizens to accept austerity measures (see Katsikas, 2012; Ardagna and Caselli, 2014). Moreover, from an electoral perspective, international negotiations have been used by the Greek government to generate the perception of being combative and active in the eyes of the Greek electorate to improve its chances of being re-elected (Katsikas, 2012).

Our setup may lead to interesting prospects for future research. First, it would be particularly interesting to study the interplay between the electoral process and citizens' welfare in an intertemporal framework to examine the interaction between reputational strengths and the diversionary mechanism. Second, formalizing a probabilistic environment would allow us to describe how the propagation of idiosyncratic shocks in the federation is affected by the diversionary channel.

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[30]In the same year, the United States also benefited from such a windfall fiscal gain, with the same effect on citizens' claims, due to strong April tax revenues (the so-called *April Surprise*), allowing a reevaluation of the primary budget surplus close to 50%.

[31]For example, Prime Minister A. Tsipras was claimed in 2016 that creditors have worsened the Greek crisis. See, e.g., https://www.theguardian.com/world/2016/sep/11/alexis-tsipras-greece-criticises-creditors-thessaloniki.

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Supplementary Material for online publication only

Appendix A. Motivating evidence

Year	Event	City	Province
1935	Regina Riot	Regina	Saskatchewan
1935	Battle of Ballantyne	Vancouver	British Columbia
1935	Unemployment Riots	Calgary	Alberta
1938	Bloody Sunday	Vancouver	British Columbia
1942	Battle of Bowmanville	Bowmanville	Ontario
1942	Arsonists fires	St. John's	Newfoundland
1944	The Montreal and Verdun ZootSuit disturbances	Montreal	Quebec
1945	The Halifax VE-Day riots	Halifax	Nova Scotia
1949	Airplane Bombing	Quebec	Quebec
1955	The Richard Riot	Montreal	Quebec
1963	Bombings by separatist Front de Libation du Quebec (FLQ)	Montreal	Quebec
1968	SaintJeanBaptiste Day riot	Montreal	Quebec
1969	Sir George Williams Computer Riot	Montreal	Quebec
1969	The Murray-Hill riot	Montreal	Quebec
1970	FLQ kidnapping - October Crisis	Montreal	Quebec
1971	Gastown Riots	Vancouver	British Columbia
1993	Stanley Cup Riot	Montreal	Quebec
1997	Teachers' Protest		Ontario
1998	Native loggers protested restrictions		New Brunswick
1999	Native People Defiance		British Columbia
2001	Quebec City Summit of the Americas	Quebec	Quebec
2001	Canada Day Riot	Edmonton	Alberta
2002	Concordia University Netanyahu riot	Montreal	Quebec
2003	The Exploited Montreal riot	Montreal	Quebec
2010	Riots in Vancouver	Vancouver	British Columbia
2010	Riots in Toronto	Toronto	Ontario
2010	Prison riot in Quebec	Quebec	Quebec
2011	Riots in Vancouver	Vancouver	British Columbia
2012	Quebec student protests	Quebec	Quebec

Table A.5: Internal Conflicts in Canada at the Provincial level: 1935-2013.

A Brief History of Canada,

http://www3.sympatico.ca/goweezer/canada/canhist.htm.

History Since Confederation, Canadian Encyclopedia,

https://www.thecanadianencyclopedia.ca/en/article/history-since-confederation.

Riots and civil disorder in Canada, Wikipedia,

https://en.wikipedia.org/wiki/Category:Riots_and_civil_disorder_in_Canada.

List of Conflicts in Canada, Wikipedia,

https://en.wikipedia.org/wiki/List_of_conflicts_in_Canada.

Appendix B. Proof of proposition 1

The vector of citizens' effort is $\mathbf{x}_2 := (x_{2,1}, \cdots, x_{2,M})$, and the vector of districts' effort is $\hat{\mathbf{y}}_2 := (\hat{y}_{2,1}, \cdots, \hat{y}_{2,N})$, where $\hat{y}_2 := e \sum_{s=1}^{M} y_{2,s}$. The Lagrange function $\mathcal{L}$ related to the maximization problem (20) is

$$\begin{aligned}\mathcal{L}(x_{2,n}, y_{2,n}, \lambda_n) = \alpha^I(\mathbf{x}_1) &\left[\tau \sum_{s=1}^{M} (1 - x_{1,s}) + \mathcal{D}\right] \\ &+ \beta\alpha^j(\mathbf{x}_2)\left[\tau \sum_{s=1}^{M}(1 - x_{2,s} - y_{2,s}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right] \\ &+ \lambda_n \left[\tau \sum_{s=1}^{M}(1 - x_{2,s} - y_{2,s}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right],\end{aligned} \tag{B.1}$$

where $\lambda_n \geq 0$ is the Lagrange multiplier of the constraint $g_2 \geq 0$. The following subsection computes the first-order conditions.

Appendix B.1. First-Order Conditions

By (B.1), the Karush-Kuhn-Tucker (KKT) conditions are

$$\frac{\partial \mathcal{L}}{\partial x_{2,n}} = \beta \frac{\partial \alpha^j(\mathbf{x}_2)}{\partial x_{2,n}} \left[\tau \sum_{s=1}^{M}(1 - x_{2,s} - y_{2,s}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right] - \beta\tau\alpha^j(\mathbf{x}_2) - \lambda_n \tau = 0, \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial y_{2,n}} = -\left[\tau + \frac{\partial \psi(\hat{\mathbf{y}}_2)}{\partial y_{2,n}}(1+r)\mathcal{D}\right]\left[\lambda_n + \beta\alpha^j(\mathbf{x_2})\right] = 0, \tag{B.3}$$

$$\lambda_n \left[\tau \sum_{s=1}^{M}(1 - x_{2,s} - y_{2,s}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right] = 0. \tag{B.4}$$

The symmetric equilibrium is characterized by $x_{2,n} =: x_2$, $y_{2,n} =: y_2$, $\lambda_n =: \lambda$, $\forall n \in \{1, \cdots, M\}$, and $\hat{y}_{2,i} = \hat{y}_2$, $\forall i \in \{1, \cdots, N\}$. Using Eqs. (10) and (11) , the symmetric equilibrium leads to

$$\frac{\partial \alpha^j(\mathbf{x}_2)}{\partial x_{2,n}} = a x_{2,n}^{a^j - 1} \left(\frac{\sum_{m \neq n}^{M} x_{2,m}^{a^j}}{\left(\sum_{m=1}^{M} x_{2,m}^{a^j}\right)^2}\right) = \frac{a^j}{x_2}\left(\frac{M-1}{M^2}\right),$$

$$\frac{\partial \psi(\hat{\mathbf{y}}_2)}{\partial y_{2,n}} = -b\hat{y}_{2,i}^{-b-1}\left(\frac{\sum_{s\neq i}^{N}\hat{y}_{2,s}^{-b}}{\left(\sum_{s=1}^{N}\hat{y}_{2,s}^{-b}\right)^2}\right) = -\frac{b}{My_2}\left(\frac{N-1}{N^2}\right).$$

Since $\psi = 1$ and $\alpha^j = 1/M$, the KKT conditions (B.2)-(B.3)-(B.4) become

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{\beta a^j}{x_2}\left(\frac{M-1}{M^2}\right)[(1-x_2-y_2)-(1+r)\mathcal{D}] - \frac{\beta\tau}{M} - \lambda\tau = 0, \tag{B.5}$$

$$\frac{\partial \mathcal{L}}{\partial y_2} = -\left[\tau - \frac{b(1+r)\mathcal{D}}{y_2 M}\left(\frac{N-1}{N^2}\right)\right]\left[\lambda + \frac{\beta}{M}\right] = 0, \tag{B.6}$$

$$\lambda\left[\tau M(1-x_2-y_2)-(1+r)\mathcal{D}\right] = 0. \tag{B.7}$$

Now, we look at the complementary slackness condition, considering two cases.

Case 1. $\lambda > 0$. Using (B.7), $\tau M(1-x_2-y_2) = (1+r)\mathcal{D}$. By substituting in (B.5), we have $\lambda = -\beta/M < 0$, which establishes a contradiction.

Case 2. $\lambda = 0$. Using (B.7), $\tau M(1-x_2-y_2) > (1+r)\mathcal{D}$, and (B.5)-(B.6) become

$$\frac{\partial \mathcal{L}}{\partial x_2} = \frac{\beta a^j}{x_2}\left(\frac{M_i-1}{M^2}\right)[\tau M(1-x_2-y_2)-(1+r)\mathcal{D}] - \frac{\beta\tau}{M} = 0,$$
$$\frac{\partial \mathcal{L}}{\partial y_2} = \frac{\beta}{M}\left[\frac{b(1+r)\mathcal{D}}{y_2 M}\left(\frac{N-1}{N^2}\right) - \tau\right] = 0,$$

hence; the unique critical-point $(\check{x}_2, \check{y}_2)$ is, using $\tilde{N} := (N-1)/N^2$,

$$\check{x}_2 = \frac{a^j(M-1)}{1+a^j(M-1)}\left[1 - \left(1+b\tilde{N}\right)\frac{(1+r)\mathcal{D}}{\tau M}\right], \tag{B.8}$$

$$\check{y}_2 = \frac{b\tilde{N}(1+r)\mathcal{D}}{\tau M}. \tag{B.9}$$

By denoting $A_i^j := a_i^j(M-1)/(1+a_i^j(M-1))$, and $B := 1+b\tilde{N}$, Eqs. (25) and (26) in the main tex immediately follow.

Finally, the couple of efforts $(\check{x}_2, \check{y}_2)$ is the unique equilibrium if and only if it is the unique global maximum on the compact-space $\mathscr{C} := \{(s,t) \in [0,1]^2;\ s+t < 1\}$, as shows the following subsection.

Appendix B.2. Second-Order Conditions

To establish the concavity of the Lagrange-function, we compute the following second-derivatives. By (B.2), and (B.3) we obtain

$$\frac{\partial^2 \mathcal{L}}{\partial (x_{2,n})^2} = \beta \frac{\partial^2 \alpha^j(\mathbf{x}_2)}{\partial (x_{2,n})^2}\left[\tau \sum_{s=1}^{M}(1 - x_{2,s} - y_{2,s}) - \psi(\hat{\mathbf{y}}_2)(1+r)\mathcal{D}\right] - 2\beta\tau \frac{\partial \alpha^j(\mathbf{x}_2)}{\partial x_{2,n}},$$

$$\begin{aligned}
\frac{\partial^2 \mathcal{L}}{\partial x_{2,n}\partial y_{2,n}} &= -\beta \frac{\partial \alpha^j(\mathbf{x}_2)}{\partial x_{2,n}}\left[\tau + \frac{\partial \psi(\hat{\mathbf{y}}_2)}{\partial y_{2,n}}(1+r)\mathcal{D}\right], \\
\frac{\partial^2 \mathcal{L}}{\partial (y_{2,n})^2} &= -\frac{\partial^2 \psi(\hat{\mathbf{y}}_2)}{\partial (y_{2,n})^2}(1+r)\mathcal{D}\left[\lambda_n + \beta\alpha^j(\mathbf{x}_2)\right], \\
\frac{\partial^2 \mathcal{L}}{\partial x_{2,n}\partial \lambda_n} &= -\tau, \\
\frac{\partial^2 \mathcal{L}}{\partial y_{2,n}\partial \lambda_n} &= -\left[\tau + \frac{\partial \psi(\hat{\mathbf{y}}_2)}{\partial y_{2,n}}(1+r)\mathcal{D}\right],
\end{aligned}$$

and, using Eq. (B.1), we have $\partial^2 \mathcal{L}/\partial(\lambda_n)^2 = 0$.

In the symmetric equilibrium, by (B.8) and (B.9), second-derivatives evaluated at the critical-point $(\check{x}_2,\, \check{y}_2)$ become

$$\left.\frac{\partial^2 \mathcal{L}}{\partial x_{2,n}\partial y_{2,n}}\right|_{(\check{x}_2,\check{y}_2)} = \left.\frac{\partial^2 \mathcal{L}}{\partial y_{2,n}\partial \lambda_n}\right|_{(\check{x}_2,\check{y}_2)} = 0,$$

since the first-order conditions lead to $\tau + \frac{\partial \psi(\hat{\mathbf{y}}_2)}{\partial y_{2,n}}(1+r)\mathcal{D} = 0$. In addition, we have

$$\begin{aligned}
\left.\frac{\partial^2 \mathcal{L}}{\partial (x_{2,n})^2}\right|_{(\check{x}_2,\check{y}_2)} = \beta \left.\frac{\partial^2 \alpha^j(\mathbf{x}_2)}{\partial (x_{2,n})^2}\right|_{\check{x}_2} &\left[\tau M(1 - \check{x}_2 - \check{y}_2) - (1+r)\mathcal{D}\right] \\
&- 2\beta\tau \left.\frac{\partial \alpha^j(\mathbf{x}_2)}{\partial x_{2,n}}\right|_{\check{x}_2} =: E_1,
\end{aligned} \tag{B.10}$$

$$\left.\frac{\partial^2 \mathcal{L}}{\partial (y_{2,n})^2}\right|_{(\check{x}_2,\check{y}_2)} = -\left.\frac{\partial^2 \psi(\hat{\mathbf{y}}_2)}{\partial (y_{2,n})^2}\right|_{\check{y}_2} (1+r)\mathcal{D}\left[\lambda + \frac{\beta}{M}\right] =: E_2. \tag{B.11}$$

Consequently, the Hessian-matrix, denoted by $\mathbf{H}(\cdot,\cdot)$, related to the Lagrange function evaluated at the critical-point $(\check{x}_2, \check{y}_2)$ is given by

$$\mathbf{H}|_{(\hat{x}_2,\hat{y}_2)} = \begin{pmatrix} E_1 & 0 & -\tau \\ 0 & E_2 & 0 \\ -\tau & 0 & 0 \end{pmatrix}.$$

As $(\alpha^j)''(\mathbf{x}_2) < 0$ and $\psi''(\hat{\mathbf{y}}_2) > 0$, we have: $E_1 < 0$, $E_2 < 0$; namely, $\text{tr}(\mathbf{H}) = E_1 + E_2 < 0$ and $\det(\mathbf{H}) = -\tau^2 E_2 > 0$. Thus, the unique critical-point $(\hat{x}_2, \hat{y}_2)$ is the unique global maximum of the Lagrange-function.

In the last step, we show that $(\check{x}_2, \check{y}_2) \in \mathscr{C}$, namely $\check{x}_2 \geq 0$, $\check{y}_2 \geq 0$, and $\check{x}_2 + \check{y}_2 \leq 1$. First, by (B.9), we have $\check{y}_2 \geq 0$, $\forall \mathcal{D} \geq 0$, and by (B.8), $\check{x}_2 \geq 0$ if and only if $\mathcal{D} \in [0, \bar{\mathcal{D}}]$, where

$$\bar{\mathcal{D}} := \frac{1}{\kappa B}. \tag{B.12}$$

Second, using Eqs. (B.8)-(B.9), the function $\zeta(\mathcal{D}) := \check{x}_2 + \check{y}_2$ is given by

$$\zeta(\mathcal{D}) = A^j - \kappa\mathcal{D}\left[1 - B(1 - A^j)\right]. \tag{B.13}$$

From (B.13), as $\zeta(\mathcal{D})$ linearly depends on $\mathcal{D}$, we can distinguish the two following cases.

(i) If ζ decreases with $\mathcal{D}$, we have, by (B.13),

$$\max_{\mathcal{D}\in[0,\bar{\mathcal{D}}]} \zeta(\mathcal{D}) = \zeta(0) = A^j < 1.$$

(ii) If ζ increases with $\mathcal{D}$, using (B.12), we have

$$\max_{\mathcal{D}\in[0,\bar{\mathcal{D}}]} \zeta(\mathcal{D}) = \zeta(\bar{\mathcal{D}}) = \frac{B-1}{B} < 1$$

Finally, for any $\mathcal{D} \in [0, \bar{\mathcal{D}}]$, the couple $(\check{x}_2, \check{y}_2)$ is the unique global maximum on $\mathscr{C}$, namely the unique equilibrium, and we note $(\check{x}_2, \check{y}_2) =: (x_2^*, y_2^*)$.

Appendix C. The incumbent's programme

By (19) and (32), the first-order condition (33) can be written: $2\beta h\tau\lambda(1+r)\phi(\mathcal{D}) = 0$, where

$$\phi(\mathcal{D}) := 2\kappa B\tilde{A}\bar{R} + \tilde{A} - \Psi - 2\kappa B\tilde{A}\mathcal{D}, \tag{C.1}$$

with $\Psi := 1/[4\beta h\tau]$, and $\bar{R} := R/[2\lambda(1+r)(B-1)]$.

Clearly, $\phi'(\mathcal{D}) = -2\kappa B\tilde{A}$, and the sign of $\tilde{A}$ determines the sign of the second-order derivative. If $\tilde{A} > 0$, ϕ is strictly decreasing, the objective function $\mathbb{E}[V]$ of the incumbent is strictly concave, and, if there is a critical point in $(0, \bar{\mathcal{D}})$, it defines the unique global maximum. If $\tilde{A} \leq 0$, in contrast, ϕ is increasing, and there is no interior maximum. Let us characterize the solution of the maximization problem in these two cases, respectively.

Strict concavity: $\tilde{A} > 0$**.** Eq. (C.1) defines the unique critical point

$$\check{\mathcal{D}} = \bar{R} + \frac{\tilde{A} - \Psi}{2\kappa B\tilde{A}}.$$

We must ensure that $\check{\mathcal{D}} \in (0, \bar{\mathcal{D}})$. First, by (C.1), we have $\phi(0) > 0$ if and only if

$$\tilde{A} > \tilde{A}_1 := \frac{\Psi}{2\kappa\bar{R}\bar{B} + 1}. \tag{C.2}$$

Second, by (C.1), we have $\phi(\bar{\mathcal{D}}) < 0$ if and only if

$$\tilde{A} < \tilde{A}_2 := \frac{\Psi}{2\kappa B(\bar{R} - \bar{\mathcal{D}}) + 1}. \tag{C.3}$$

We assume that $\bar{R} > \bar{\mathcal{D}}$, namely $R > 2\lambda(1+r)(B-1)\bar{\mathcal{D}} \Leftrightarrow R > 2\lambda\tau M(B-1)/B$.

Consequently, if $\tilde{A} > 0$, the objective function ($\mathbb{E}[V]$) is strictly concave, the first-order derivative is a decreasing continuous function, and the maximum $\mathcal{D}^*$ in $[0, \bar{\mathcal{D}}]$ is characterized by the following three cases.

i. If $\tilde{A} \leq \tilde{A}_1$, $\phi(0) \leq 0$, and $\mathcal{D}^* = 0$.

ii. If $\tilde{A} \geq \tilde{A}_2$, $\phi(\bar{\mathcal{D}}) \geq 0$, and $\mathcal{D}^* = \bar{\mathcal{D}}$.

iii. If $\tilde{A}_1 < \tilde{A} < \tilde{A}_2$, we have $\mathcal{D}^* = \check{\mathcal{D}} \in (0, \bar{\mathcal{D}})$.

Convexity: $\tilde{A} \leq 0$. In this case, the objective function ($\mathbb{E}[V]$) is convex, the first-order derivative (ϕ) is a continuous and increasing function, and the global maximum is reached at the bound, namely $\mathcal{D}^* \in \{0, \bar{\mathcal{D}}\}$. Therefore, as $\tilde{A} < 0 < \tilde{A}_1 < \tilde{A}_2$, it follows that $\phi(0) < 0$ and $\phi(\bar{\mathcal{D}}) < 0$, hence $\mathcal{D}^* = 0$.

Equilibrium Characterization: Summing up, we have:

- if $\tilde{A} \leq \tilde{A}_1 \Rightarrow \mathcal{D}^* = 0$,
- if $\tilde{A}_1 < \tilde{A} < \tilde{A}_2 \Rightarrow \mathcal{D}^* = \bar{R} + \frac{\tilde{A}-\Psi}{2\kappa B\tilde{A}} \in (0, \bar{\mathcal{D}})$,
- if $\tilde{A} \geq \tilde{A}_2 \Rightarrow \mathcal{D}^* = \mathcal{D}$.

Appendix D. Social welfare

Let $\beta = 1/(1+r)$. Using Eq. (36), the objective of the social planner $\mathcal{D} \mapsto \mathbb{E}[W](\mathcal{D})$ is a quadratic function, and describes a U-shaped curve. The solution of the maximization problem is

$$\max_{\mathcal{D}\in[0,\bar{\mathcal{D}}]} \mathbb{E}[W] = \max\left\{\mathbb{E}[W](0)\ ;\ \mathbb{E}[W](\bar{\mathcal{D}})\right\}.$$

From Eq. (36), we obtain, using (25),(26), (28)

$$\mathbb{E}[W](0) = -2\tau M\beta\tilde{A}\left[\frac{1}{2} - 2h\tau\beta\tilde{A}\right] + \beta\tau M(1-A^O) + \tau M(1-A^I), \tag{D.1}$$

$$\mathbb{E}[W](\bar{\mathcal{D}}) = \frac{\beta\tau M}{B} + \tau M\left[1 - A^I - \frac{\beta A^I}{B}\right]. \tag{D.2}$$

Let us introduce $\Phi(B) := \mathbb{E}[W](\bar{\mathcal{D}}) - \mathbb{E}[W](0)$, for $B \in [1,+\infty)$. First, we have $\Phi'(B) = \partial\mathbb{E}[W](\bar{\mathcal{D}})/\partial B < 0$, as $A^I < 0$. Second, if $\tilde{A} > 0$, we have

$$\Phi(0) = -2\tau M\beta\tilde{A}\left[\frac{1}{2} + 2h\tau\beta\tilde{A}\right] < 0,$$

hence $\Phi(B) < 0$, for any $B \geq 1$. Consequently, $\mathbb{E}[W](\bar{\mathcal{D}}) < \mathbb{E}[W](0)$, and $\mathcal{D}^{SW} = 0$.

Third, if $\tilde{A} < 0$, we have $\Phi(0) > 0$, and

$$\lim_{B\to\infty} \Phi(B) = 2\tau M\beta\tilde{A}\left[\frac{1}{2} - 2h\tau\beta\tilde{A}\right] - \beta\tau M(1-A^O) < 0.$$

According to the Intermediate Value Theorem, there is a critical value $\bar{B} > 1$, such that: $\phi(B) > 0 \Leftrightarrow \mathbb{E}[W](\bar{\mathcal{D}}) > \mathbb{E}[W](0)$ for $B < \bar{B}$; and $\Phi(B) \leq 0 \Leftrightarrow \mathbb{E}[W](\bar{\mathcal{D}}) \leq \mathbb{E}[W](0)$ for $B \geq \bar{B}$.