Document de Recherche

n° 2006-37

« Multi-Period Health Insurance Contracts and Bayesian Updating of Beliefs »

Mouhamadou FALL
Anne LAVIGNE

Laboratoire d'Economie d'Orléans – UMR CNRS 6221 Faculté de Droit, d'Economie et de Gestion,
Rue de Blois, B.P. 6739 – 45067 Orléans Cedex 2 - France
Tél : 33 (0)2 38 41 70 37 – 33 (0)2 38 49 48 19 – Fax : 33 (0)2 38 41 73 80
E-mail : leo@univ-orleans.fr - http://www.univ-orleans.fr/DEG/LEO
Multi-period Health Insurance Contracts and Bayesian Updating of Beliefs

Mouhamadou Fall and Anne Lavigne

31 March 2006

Preliminary version submitted to the scientific committee of the EGRIE Conference, Barcelona, September 2006

Abstract

Informational asymmetries can be overcome by offering dynamic contracts with experience rating. Among others, Cooper and Hayes (1987) have theoretically demonstrated this result in both a monopolistic and a competitive environment. The purpose of this contribution is to extend the Cooper and Hayes' model to take into account the Bayesian updating of beliefs by insurers. It is motivated by the observed fact that, in health insurance markets, some genetic diseases have a higher probability of recurrence for gene carriers than for non carriers. We show that, in a two-period monopolistic setting, the updating of beliefs in the second period is detrimental to the high-risk individuals that have supported a disease in the first period: these individuals can no longer benefit from full insurance as they did in the Cooper and Hayes' setting. When allowing entrants, experience rating is no longer available to supply contracts to high-risk individuals. Competition is still beneficial to all individuals but detrimental to the incumbent company that is bound to forsake its profits to keep its clientele.

JEL Classification: G220, D820

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1 Laboratoire d'économie d'Orléans, UMR 6221, Rue de Blois, BP 6739, 45067 Orléans Cedex 2, France. Corresponding author: Anne.Lavigne@univ-orleans.fr
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1. Introduction

Informational asymmetries are known to plague insurance markets. Insurance companies have developed contractual techniques to overcome this nuisance. In static contracts, incentive-compatible mechanisms rely on discriminating pricing and coverage. Insurers typically diversify their contractual supply through menus of contracts in which premium and coverage is positively correlated so that the more risky individuals choose a contract with a high premium and an extended coverage, and the less risky opt for a low premium and a partial coverage. This menu diversification enables a risk pooling of homogeneous classes of risks, the fair pricing principle being preserved in a competitive environment: in each class of risks, premiums are set at their actuarial level so that each contract provides no profit to insurance companies at competitive equilibrium.

However single-period contracts do not allow insurance companies to exploit all the relevant information needed to set premium and coverage at their best level. On the one hand, past accident experience can be useful to give incentives to insured to reveal their prior risk characteristics, and to undertake actions that can reduce their probability of accident on the other hand. An extensive set of theoretical models has shown that experience rating is an efficient mean to overcome adverse selection (see Dionne and al., 2000) and moral hazard (see Winter, 2000). On an empirical ground, experience rating is widely used in many insurance markets, such as car insurance through no-claim discount pricing. Using multi-period contracts can mitigate the pervasive consequences of adverse selection and moral hazard.

Beyond non-life insurance markets, multi-period contracts are also used in insurance for banking loans, in life insurance and in health insurance. If moral hazard is seldom observed in these markets\(^2\), progress in genetics enhances adverse selection phenomena since potential subscribers may have access to private genetic information on their probability to develop a given pathology. The adverse selection problem is likely to be exacerbated when genetic testing is accurate and when pathology is

\(^2\) Even uncommon it may exists, insurance coverage giving incentives to risky actions leading to benign pathologies for example.
monogenic\textsuperscript{3}. Multi-period contracts are therefore of interest in health insurance, for monogenetic diseases, but also for polygenic diseases in which genetic factors are combined to environmental and behavioural factors.

Some genetic diseases have a remarkable property which distinguishes them from other risks. In car insurance for instance, accidents are serially uncorrelated: the annual probability of having a car accident does not change through time for a given individual. It is true that experience may improve driving capacities, therefore reducing accident probabilities as time elapses. This argument is used to charge higher premiums to young drivers when they first apply for an insurance policy. But after a couple of years of experience, the positive effects of learning are dissipated. Abstracting from moral hazard, the annual probability of accident remains constant, at least up to a certain age at which driving abilities decline and expose drivers to accrued risks of accident. The seminal paper by Cooper and Hayes (1987) is based on the assumption of a constant probability of accident through time, which remains private information for the policyholder. The only revealed information is the occurrence (or not) of an accident in each period.

On the opposite, in some polygenic diseases, a pathologic episode at a given point of time increases the probability of recurrence through time: the presence of an altered gene increases the probability of recurrence for gene carriers, with respect to non-carriers who may sporadically develop the disease (Seynaeve and al., 2004; Kirova and al., 2005). Breast cancer is a relevant illustration. In developed countries, about one woman in nine develops breast cancer in her lifetime, and one in forty dies from it. Medical research has shown that mutations of BRCA1 and BRCA2 genes are associated with an enhanced risk of breast cancer. About 5\% of breast cancers are the results of a gene mutation (BRCA1 or BRCA2). Still, 95\% of breast cancers are not inherited, resulting from environmental and behavioural factors (diet, lifestyles, social exposures, birth control pill taking, hormone replacement therapy...). In a medical contribution, Kirova and al. (2005) examined whether mutation status influenced the rate of breast cancer recurrence following breast-conserving surgery and radiotherapy. They showed that no significant differences in breast cancer recurrence as first event between carriers and non-carriers,. However the rate of contralateral breast cancer was significantly increased for BRCA1/2 carriers vs. non-carriers. The same kind of observations can be found for ovarian cancers, prostatic cancers, and even for mental pathologies such as suicides.

\textsuperscript{3} A monogenic disease is linked to a specific mutation of a gene for which the presence of an altered gene makes the future exposure to the disease nearly certain. The Huntington Chorea, a neurodegenerative disease, is an illustrative example. For a presentation of genetic diseases and their insurability, see Fall, 2004.
These empirical observations suggest that past morbid episodes provide not only information on prior risk characteristics of policyholders for a given insurance company, revealed by their choice in a menu of contracts as in the Cooper and Hayes’ model, but also information on their future probabilities of developing the disease. Therefore insurers are likely to exploit this double informational rent in a competitive environment.

The purpose of our contribution is precisely to analyse the supply of multi-period contracts when potential health insurance subscribers have private genetic information on diseases with increasing recurrence probabilities when insurers update their beliefs on disease probabilities. The rest of the paper is organised as follows. The main assumptions are presented and justified in section 2. Section 3 is devoted to the monopolistic setting, while section 4 deals with the competitive setting. Section 5 concludes and suggests possible extensions for further research.

2. Assumptions and basic model

2.1 Assumptions on information

Let us consider a large population of individuals which are submitted to a mandatory genetic test. This test reveals the predisposition to a polygenic disease (say, breast cancer) and therefore a prior probability to develop the disease in the future.

The result of the test reveals two prior possible types: "being a low risk", which leads to a probability \( p_L \) to be exposed to the disease and a probability \( 1-p_L \) not to develop the disease; "being a high risk" which expose to the disease with a probability \( p_H \) and which does not with a probability \( 1-p_H \).

We assume that \( 0 < p_L < p_H < 1 \) and we denote \( \lambda \) (respectively \( 1-\lambda \) ) the proportion of low-risk individuals (respectively high-risk). These proportions are assumed to be common knowledge and stable through time. The information revealed by the test is private.

In a multi-period setting, informational asymmetry does not only concern types (the genetic predisposition to develop the disease), but also the past history of effective pathologic episodes. When only one insurer provides insurance contracts, he gets information on the pathologic episodes of his policyholders. We assume that if a subscriber has chosen a contract designed for low-risk individuals (those with a prior \( p_L \) probability to develop the disease) in the first period, and if he develops the
disease in this first period, the insurer revises his belief through Bayesian updating so that the posterior probability assigned by the insurer is lower than \( p_i \). On the opposite, if the insured has chosen a contract designed for high-risk individuals in the first period, but has not suffered from the disease in that period, the insurer revises his belief so that the posterior probability assigned by the insurer is higher than \( p_{ni} \). In other words, for the insurer, the genetic predisposition together with a pathologic episode increases the recurrence probability. On the contrary, absent genetic predisposition, a pathologic episode in the first period signals a sporadic disease and therefore a future lower probability of illness. Assuming a Bayesian update of insurer’s beliefs is the main difference in our setting, compared to Cooper & Hayes (1987).

When considering a competitive insurance market, the informational asymmetry on the types (low- or high-risk) cumulates with an informational asymmetry between the insurers on morbid episodes. Three competitive settings are feasible with multi-period contracts: (i) full commitment in which both insurer and insured are committed to enforce the contract in each period without any possibility to breach the contract; (ii) partial (or semi-) commitment in which the insurer cannot renegotiate the contract while the insured may do so at the end of each period; (iii) no commitment in which both the insurer and the insured may breach the contract. A full commitment contract has the same features as the monopolistic contract and the no-commitment situation is not different from the mere repetition of static (single-period) contracts. In these first two cases, the knowledge of morbid episodes does not bring any competitive informational advantage: the monopoly does not share its informational rent in the first case, and the information on morbid episodes has no value in the second case. More interesting is the partial commitment situation, characterised by the binding supply of an installed company which a policyholder may quit at the end of each period, notably if a rival enters the market with a better supply. In this intermediary case we assume that the installed company has superior information with respect to the potential entrant: it knows the contract chosen by the insured in the first period on the one hand and his effective morbid history in the first period on the other hand. With this information the installed company may update its prior beliefs on recurrence probabilities of disease of its own clientele. We assume that this is not the case for a potential entrant. It may therefore assess its gains to enter the market on the prior probabilities of disease.

4 The situation in which the insured is bounded and the insurer lacks empirical relevance and is therefore not considered.

5 On the French car insurance market, this assumption could not be held since the insured are legally bounded to give the history of their past accidents (the bonus-malus coefficient at least) whenever they move to a new insurance company. They are nevertheless not compelled to indicate what their previous contract was.
In what follows and for the sake of analytical tractability, we only consider mono or multi-period contracts. We conjecture that the extension of our modelling to an infinite number of periods does not qualitatively modify our results (see Cooper and Hayes (1987) for a justification of the argument).

2.2 Preferences of insurer(s) and insureds

Let us assume that individuals are endowed with an initial wealth $w$ and are exposed to a monetary loss $D$ when they suffer from a potentially genetic disease with a prior probability $\rho_i (i = L,H)$ depending on their type, low (L) or high (H) risk. Policyholders' preferences are represented by a Von Neumann Morgenstern utility function. The utility function $U$ is defined in the wealth space and is twice differentiable with the standard assumptions of monotonicity and strict risk aversion $U''(w) < 0 < U'(w)$.

The insurer either a monopoly or a competitive company is assumed to be risk neutral. A one-period contract is a pair $(\alpha_i, \beta_i)$ characterised by a premium $\alpha_i$ paid by an insured of type $i$ and coverage (net of premium) $\beta_i$ whenever a disease occurs. A two-period contract is a vector $(\alpha_i, \beta_i, \beta_{i,S}, \alpha_{i,N}, \beta_{i,N})$ where $\alpha_i$ and $\beta_i$ respectively denote the premium and the net indemnity of the first period contract; $\alpha_{i,S}$ and $\beta_{i,S}$ respectively denote the premium and the net compensation in the second period contingent on an effective morbid episode occurred in the first period. Symmetrically, $\alpha_{i,N}$ and $\beta_{i,N}$ denote the same variables contingent on the absence of disease in the first period. For the sake of simplicity, we denote $c_i = (\alpha_i, \beta_i)$ the first period contract and $c_{i}^{2} = \{\alpha_{i,S}, \alpha_{i,N}, \beta_{i,S}, \beta_{i,N}\}$ the second period contract with $c_{i,S} = (\alpha_{i,S}, \beta_{i,S})$ and $c_{i,N} = (\alpha_{i,N}, \beta_{i,N})$.

3. Monopolistic setting

In this section we consider a monopolistic setting in which an insurance company has asymmetric information on the true type of the insured. We first analyse the (standard) equilibrium with single-period contracts and then turn to the equilibrium with two-period contracts.

3.1 Equilibrium with single-period contracts

This setting has been analysed by Stiglitz (1977). Market equilibrium is defined as a set of contracts that maximise the insurer expected profit under participating and self-selection constraints:
Proposition (Stiglitz, 1977)

There exists a separating market equilibrium at which the insurance company supplies two different contracts for high-risk and low-risk individuals with the following properties:

(i) High-risk individuals get full insurance with a binding self-selection constraint and an unbinding participating constraint;

(ii) Low-risk individuals obtain partial insurance with a binding participating constraint and an unbinding self-selection constraint;

(iii) For the low-risk individuals to buy a strictly positive amount of coverage there must exist a sufficient proportion of high-risk individuals with respect to low-risk ones;

(iv) Monopoly makes no profit on the high-risk segment of clientele, but gets positive profits on low-risks individuals whose total surplus is extracted.

Proof: see Stiglitz (1977, pp. 418-422)

\[
\begin{aligned}
\text{Max } & E \pi(\alpha, \beta) = \sum_{i=1}^{N_i} ((1-p_i) \alpha_i - p_i \beta_i) \\
\text{st} & EU(\alpha_i, \beta_i | p_i) = (1-p_i)U(W - \alpha_i) + p_iU(W - D + \beta_i) \geq EU(0,0 | p_i) = (1-p_i)U(W) + p_iU(W - D) \quad i = L, H \\
& EU(\alpha_i, \beta_i | p_i) \geq EU(\alpha_j, \beta_j | p_j) \quad i, j = L, H \quad i \neq j
\end{aligned}
\]

3.2 Equilibrium with two-period contracts

Let us now assume that an insurance monopoly offers two-period contracts. We assume that individuals are uncommitted to their initial contract in the second period but they cannot get another
contract (leaving the insurance contract means leaving the insurance market; leaving the initial contract to switch to a rival insurer will be analysed in the following section). We first recall the Cooper and Hayes (1987) results obtained in a framework in which the insurer offers contingent contracts based on types and history of morbid events without updating his prior on probabilities of disease of his clientele. We then turn to a setting in which the insurer updates his prior beliefs to take into account the different serial correlation between morbid episodes of low and high-risk individuals.

3.2.1. Two-period contracts without Bayesian updating of beliefs

The framework considered in this sub-section has been analysed by Cooper and Hayes (1987). Market equilibrium derives from:

\[
\begin{align*}
\max_{c_s,c_j} E \pi(\alpha_i, \beta_j, \alpha_{s}, \beta_{s}, \alpha_{n}, \beta_{n}) &= \sum_{i=1}^{N_l} [(1-p)\alpha_i - p\beta_i] \\
&+ \sum_{i=1}^{N_h} [p((1-p)\alpha_{s} - p\beta_{s}) + (1-p)((1-p)\alpha_{n} - p\beta_{n})]
\end{align*}
\]

(P2)

s.t. \[
\begin{align*}
EU(\alpha_j, \beta_j, \alpha_{s}, \beta_{s}, \alpha_{n}, \beta_{n}, p_i) &\geq 2\bar{U}_i \text{ for } i = L, H \\
EU(\alpha_j, \beta_j, \alpha_{s}, \beta_{s}, \alpha_{n}, \beta_{n}, p_i) &\geq EU(\alpha_j, \beta_j, \alpha_{s}, \beta_{s}, \alpha_{n}, \beta_{n}, p_i) \text{ for } i, j = L, H, i \neq j
\end{align*}
\]

We assume no discounting for both the insurer and the insured. In the maximised function, the first term represents the expected profit of the first period, and the second one the expected profit of the second period contingent on a loss event in the first period. The first set of constraints is the set of participating constraints of type \(i\) individuals, and the second set represents the self-selection constraints. \(\bar{U}_i\) denotes the reservation utility (below this level, individuals self insure).

Proposition (Cooper and Hayes, 1987)

Market equilibrium with two-sided commitment, private information and contracts contingent on loss events in the first period is separating. The two-period equilibrium contracts have the following properties:

(i) Premium and coverage in the first period are identical to those of the single-period separating optimal contracts;

(ii) Equilibrium contracts sold to high-risk individuals lead to full insurance in each period, no experience rating and zero profit to the insurer;

(iii) Equilibrium contracts designed for low-risk individuals are contingent on loss events so that \(\hat{\alpha}_{L} < \hat{\alpha}_{S} < \hat{\alpha}_{I}\) and \(\hat{\beta}_{L} > \hat{\beta}_{S} > \hat{\beta}_{I}\) (where \(\hat{\alpha}_{i}\) and \(\hat{\beta}_{i}\) are the premium and coverage of the optimal static contracts;
(iv) Profits from equilibrium contracts sold to low-risk individuals with no loss event in the first period are positive, while profits made on the contracts designed for low-risk individuals having suffered from a morbid episode in the first period are negative.

Proof: see Cooper and Hayes (1987, pp. 402-403))

Let us now assume that the unique insurer exploits the additional information revealed by the effective occurrence of a disease in the first period by updating his belief of the occurrence of the disease in the second period. We assume that the policyholder does not proceed to the same Bayesian inference. The prospect theory developed by Kahneman and Tversky (1979) provides a first justification. Agents exposed to a given risk misperceive the objective probability distribution, overestimating lower probabilities and underestimating the highest ones. A second argument derives from limited rationality and limited ability to process information: individuals should ex ante maximise their expected utility, integrating the second period risk increase following a (hypothetical) occurrence of a disease in the first period. Such a mental accounting is assumed to be too far-sighted.

Let us denote \( q_{iS} \) the revised probability that a type \( i (i = H, L) \) develops the disease in the second period conditionally on the occurrence of a morbid episode in the first period; symmetrically \( q_{iN} \)
is the probability that a type \( i \) individual experiences a morbid episode in the second period conditionally on a non-exposure in the first period. Opposite to the "type" notion that concerns genetic characteristics, we refer to the "group" notion to differentiate individuals exposed or not to the disease. Therefore the monopoly is unaware of the true type but knows the group of a given individual since the disease exposure is assumed to be observable at no cost or error. We denote \( N_{is} \) (resp. \( N_{iw} \)) the number of type \( i \) individuals exposed (resp. unexposed) to the disease in the first period.

We assume that the occurrence of the disease for type \( L \) individuals (the gene non-carriers) in the first period leads to a *smaller* probability of recurrence in period two. This assumption is supported by medical empirical studies: some polygenic disease can sporadically develop among gene non-carriers so that recurrence is less probable than for gene carriers. On the opposite, type \( H \) individuals (the gene carriers) have their recurrence probability *increased* in the second period. Therefore \( q_{LS} = k_L p_i \) and \( q_{HS} = k_H p_i \), with \( k_L < 1 < k_H \). Moreover we assume that all individuals, whether gene carriers or not, who have not been exposed to the disease in the first period, withhold their prior probability: \( q_{LN} = p_i \) and \( q_{HN} = p_i \). With these initial assumptions, it can be easily shown that \( q_{LS} < q_{LN} < q_{HN} < q_{HS} \). Finally we assume that the individuals have not sufficient information to revise their prior probability distribution of loss (Andersson, 2001).

First period equilibrium contracts not being contingent on losses, they are identical to the equilibrium contracts of the preceding optimisation program. On the other hand, the resolution of the second period program is modified. Since the insurer knows in the second period whether individuals have been, or not, exposed to the disease in the first period, even if he is unaware of their true type, he offers a menu of contracts to each group that maximises his expected profit subject to participating and self-selection constraints.

For the insured having developed the disease in period 1, the equilibrium contracts in period two solve the following program:

\[
\begin{align*}
\text{Max} & \quad N_{HS} \left[ (1 - q_{HS}) \alpha_{HS} - q_{HS} \beta_{HS} \right] + N_{LS} \left[ (1 - q_{LS}) \alpha_{LS} - q_{LS} \beta_{LS} \right] \\
\text{s.t.} & \quad \begin{cases}
EU(\alpha_{HS}, \beta_{HS}, p_i) \geq EU(0,0, p_i) \\
EU(\alpha_{LS}, \beta_{LS}, p_i) \geq EU(0,0, p_i) \\
EU(\alpha_{HS}, \beta_{HS}, p_H) \geq EU(\alpha_{LS}, \beta_{LS}, p_H) \\
EU(\alpha_{LS}, \beta_{LS}, p_L) \geq EU(\alpha_{HS}, \beta_{HS}, p_L)
\end{cases}
\end{align*}
\]
Symmetrically for those individuals who have not been exposed to the disease in period one, the equilibrium contracts in period two solve the following program:

\[ \max_{c_{x_{1}},c_{x_{2}}} N_{x_{1}}[(1-q_{x_{1}})\alpha_{x_{1}}^N-q_{x_{1}}\beta_{x_{1}}^N] + N_{x_{2}}[(1-q_{x_{2}})\alpha_{x_{2}}^N-q_{x_{2}}\beta_{x_{2}}^N] \]

\[
\begin{align*}
\text{(P4)} & \quad \text{s.t.} \\
& \quad EU(\alpha_{x_{1}}, \beta_{x_{1}}, p_{r_{n}}) \geq EU(0,0,p_{r_{n}}) \\
& \quad EU(\alpha_{x_{2}}, \beta_{x_{2}}, p_{r_{n}}) \geq EU(0,0,p_{r_{n}}) \\
& \quad EU(\alpha_{x_{1}}, \beta_{x_{1}}, p_{r_{n}}) \geq EU(\alpha_{x_{2}}, \beta_{x_{2}}, p_{r_{n}}) \\
& \quad EU(\alpha_{x_{2}}, \beta_{x_{2}}, p_{r_{n}}) \geq EU(\alpha_{x_{1}}, \beta_{x_{1}}, p_{r_{n}})
\end{align*}
\]

**Proposition 1**

Under two-sided full commitment, private information, experience rated contracts and updating beliefs by a monopolistic insurer, the market equilibrium is separating with a menu of six contracts \((c_{x_{1}}, c_{x_{2}}, c_{x_{1}}^*, c_{x_{2}}^*, c_{x_{1}}^*, c_{x_{2}}^*)\). The two-period contracts have the following properties:

(i) Premium and coverage in the first period are identical to those of the single-period separating optimal contracts;

(ii) The second period equilibrium contract for high-risk individuals with no morbid episode in the first period is not experience-rated, leads to complete insurance and to zero profit for the insurer;

(iii) The second period equilibrium contract for low-risk individuals with no morbid episode in the first period is experience-rated, leads to the same partial coverage as the non contingent contract of the first period and to a positive profit for the insurer;

(iv) The second period equilibrium contract for high-risk individuals with morbid episode in the first period is experience-rated, leads to partial insurance and to zero profit for the insurer;

(v) The second period equilibrium contract for low-risk individuals with morbid episode in the first period is experience-rated, leads to the partial coverage and to a positive profit for the insurer;

Proof: see appendix.

These results show that information acquisition on morbid episodes enable the insurer to adapt his menu of contracts for those individuals with an effective exposure to illness in the first period.
3.3. Comparison of the Cooper and Hayes’ solution with the “updating of beliefs solution”

With respect to the Cooper and Hayes’ solution, low-risk individuals are not the only agents to suffer from the negative externality imposed by the high-risk individuals. Only the high-risk individuals with no morbid episode in the first period have their situation unchanged when the insurer updates his beliefs on the loss distribution and only these agents exert a negative externality on all the other insured groups. The high-risk individuals experiencing a disease in the first period see their situation worsened since they cannot benefit from complete coverage any longer. Low-risk agents experience a more ambiguous situation: those without morbid episode in period one have the same welfare as in the Cooper and Hayes’ solution. On the opposite low-risk agents developing a disease are priced a lower premium and get a lower coverage (since the insurer rightly believes that the disease is sporadic). It should be stressed that on average the expected utility of the low-risk individuals is unchanged with respect to the Cooper and Hayes’ solution.
4. Competitive setting

Let us now turn to a competitive insurance market. More precisely let us assume that an incumbent insurer commits to supply two-period contracts. At the end of the first period his insureds can costlessly switch to another insurer (the entrant) who supplies only single-period contracts. We assume the disease occurrence in period one is private information for both the insureds and the incumbent company; the entrant is unaware of neither the potential client's type nor his morbid status. It is likely that the incumbent insurer cannot extract his previous informational monopoly rent: he has to face the cream-skimming attempts of the entrant companies. Intuitively, for the ill low-risk individuals and the healthy high-risk individuals, the incumbent company and the entrant will supply the same menu of contracts. Their actuarial pricing are indeed equivalent since the incumbent will not update its beliefs. On the contrary an ill high-risk individual will be tempted to quit the incumbent company if a lower premium at an entrant company is expected (a high-risk individual expects the entrant to be less informed than the incumbent). Besides the entrant's objective is to attract the consumers on which the incumbent extracts a positive surplus in a monopoly situation.

4.1. Market equilibrium and properties

We define a market equilibrium as a Stackelberg equilibrium in which the incumbent company acts as a leader and the entrant as a follower. Equilibrium is solved by backward induction. In a first step we derive the optimal strategy of the entrant. In a second step, given the optimal strategy of the entrant that determines the no-switching constraints, we derive the optimal strategy of the incumbent.

4.1.1. The entrant's optimal strategy

Let $\bar{\xi} = (\bar{\alpha}, \bar{\beta})$ be the single-period contract supplied by the entrant to an (unknown) type $i$ individual in the second period. This contract obviously cannot not be contingent on the loss status since the entrant is unable to sort ill and healthy individuals. The entrant offers a pair of contracts that maximise the expected utility of type $i$ agents subject to self-selection and zero-profit constraints. We assume that participating constraints are binding since the insureds, whatever their type, always have an incentive to quit the incumbent company that extracts their surplus ceteris paribus. Entrant's program is thus:
Solving this program leads to the Rothschild and Stiglitz’s solution (1976): the entrant company offers a pair of separating contracts at fair odds based on prior types with full insurance to high-risks and partial insurance for low-risks whatever their (unknown) morbid status.

4.2.2. The optimal strategy of the incumbent company

Given the optimal strategy of the potential rivals, the incumbent company designs optimal contracts subject to a no-switching constraint. To do so it has to forsake the positive profits made in a monopolistic setting. Let \( c_{\text{HV}} \) (resp. \( c_{\text{LV}} \)) denote the second period contract designed for the healthy high- (resp. low-) risks. Symmetrically let \( c_{\text{HV}i} \) (resp. \( c_{\text{LV}i} \)) denote the second period contract designed for the high- (resp. low-) risks that have experienced a morbid episode in the first period. Unlike the entrant, the incumbent company updates its beliefs on the loss distribution to price its contracts, and sort the insureds on their observed morbidity in the first period.

Knowing the entrant’s optimal strategy, the incumbent designs a set of contracts that lead at least to the same expected utility level as the entrant’s offer otherwise its insureds would switch to a rival. The participating constraint is thus the no-cream-off constraint. As the entrant, the incumbent is bounded by a zero profit constraint.

For the (known) individuals without morbid episode in the first period, the optimal contracts \( c_{\text{HV}} \), \( c_{\text{LV}} \), supplied by the incumbent company solve for:

\[
\begin{align*}
\text{Max} \quad & EU(\alpha_i, \beta_i, \rho_i) = \rho_i u(w - D_i) + (1 - \rho_i) u(w - \alpha_i) \\
\text{s.t.} \quad & EU(\alpha_i, \beta_i, \rho_i) \geq EU(\alpha_j, \beta_j, \rho_j) \text{ for } i, j = H, L \text{ et } i \neq j \\
& (1 - \rho_i) \alpha_i - \rho_i \beta_i = 0
\end{align*}
\]

(P5)

For the (known) ill individuals, unlike the entrant, the incumbent company updates its beliefs in order to take into account all the relevant information at its disposal for the whole time-span of the
contract. This updating could however be detrimental since the rivals have the same expectations as the individuals about the loss distributions in the second period. In other words, while the entrant’s expected profit is based on the prior probability $p$, the expected profit of the installed company is based on the distorted probability, i.e. $q_s$. The incumbent is thus not able to maximise its expected profit due to the presence of rivals on the market and its optimal strategy solves for:

$$\max_{(\alpha^*_s, \beta^*_s, p_s)} \text{EU}(\alpha^*_s, \beta^*_s, p_s) = p_s u(w - D + \beta^*_s) + (1 - p_s) u(w - \alpha^*_s)$$

s.t.

$$(1 - q_s)\alpha^*_s - q_s \beta^*_s = 0$$

The incumbent looks for optimal contracts $c^*_rs$ and $c^*_ls$ that maximise the expected utility of those insureds that have experienced a disease in period one. Its concern is to avoid that the ill individuals, notably the ex-ante high-risks, switch to rivals. This gives a justification for the non-cream-skimming constraint which, if binding, means that the insured is indifferent between staying at the incumbent and switching to a rival. This last constraint must be compatible with the self-selection and zero-profit constraints.

**Proposition 2**

Under informational asymmetry, contracts contingent on loss history, updating of beliefs by the incumbent insurer and no-updating by rivals, the competitive market equilibrium is separating and made of a set of five contracts $(c^*_H, c^*_L, c^*_Hw, c^*_Lw, c^*_Hr, c^*_Lr)$ displaying the following properties:

(i) First-period premium and coverage are identical to the ones of separating competitive one-period contracts;

(ii) Healthy high-risks randomly choose between the incumbent company and a rival in the second period and get full insurance at fair odds;

(iii) Healthy low-risks randomly choose between the incumbent company and a rival in the second period and get partial insurance at fair odds;

(iv) The incumbent does not supply contracts to ill high-risks who switch to a rival company that provide them with complete insurance; in the second period; the ill high-risks’ self-selection constraint is binding; their no-cream-off constraint is incompatible with the self-selection constraint and the zero-profit constraint;

(v) The ill low-risks stay at the incumbent company, get partial insurance and their no-cream-off constraint is binding;

(vi) Potential entry of rival companies is sufficient for the incumbent company not to design an experience-rated strategy for high-risks; experience rating is only applicable to low-risks.

Proof: see appendix.
Comments:

For the group that has not been exposed to the disease in the first period, the incumbent company falls into line with the optimal contracts designed by the potential rival. This behaviour is sound since the installed company does not update its beliefs. Compared to the monopolistic setting, the only difference is the nil profit made on the healthy low-risks of the first period. In other words the incumbent company cannot extract the surplus of these individuals in order to prevent the entry of a rival company. This situation benefits to the policyholders since they keep their surplus while getting coverage.

For the group having developed a disease in the first period, compared to the monopolistic setting, the high-risks leave the installed company. On the opposite the low risks stay have no incentive to switch to a rival because they know they can get a more favourable pricing at the incumbent company which infers that have sporadically, and not genetically, developed the disease.

Figure 4: Competitive equilibrium two-period contracts
4.2. Comparison with the Cooper and Hayes' solution (1987)

Updating of beliefs and entry of rival companies on the insurance market are beneficial to the policyholders. High-risks that have a morbid episode in the first period can always get full insurance. The presence of rival companies annihilates the possibility of experience rating by the incumbent company. From this point of view our results are close to those of Cooper and Hayes but for a qualification. Full insurance and improvement of high-risks' welfare are only due to the potential entry of rivals while in Cooper and Hayes' model the existence of a potential rivalry leaves the ill high-risks' welfare unchanged. On the opposite the healthy high-risks do not gain from the entry of potential rival companies. Their utility is the same as in the monopolistic setting and they still get full insurance. Eventually, compared to the Cooper and Hayes' model, entry of rival companies is ex-ante Pareto-improving for all high-risks whatever their morbid history in the first period.

As regards low-risks, entry of rival companies benefits to all policyholders whatever their morbid episode in the first period. Their welfare is enhanced compared to the monopoly setting since the incumbent company no longer extracts their surplus: the latter cannot get positive profits on contracts sold on low-risks (whatever their health status in period one) because it has to modify its pricing-coverage menu so that the policyholders do not switch to a rival company. This new offer leads to a nil profit on both groups (ill and healthy) of low-risks. In terms of welfare, the healthy policyholders have nevertheless a better position compared to the Cooper and Hayes' model.

5. Conclusion and extensions

In the presence of informational asymmetries, acquisition of genetic information might jeopardise the health insurance market. Through genetic testing potential policyholders might get private information on their health status, i.e. their probability to be exposed to a given genetic disease which might lead to adverse selection problems and market failures absent proper incentives designed by insurance companies. In their two-period model, Cooper and Hayes (1987) show that the design of separating dynamic contracts can overcome adverse selection problems. Their main result is that at market equilibrium high-risk individuals (those with a high probability of accident) get full insurance with no experience-rating whereas low-risk individuals (those with a low probability of accident) have partial insurance and are experience-rated.
This purpose of our contribution is to extend the Cooper and Hayes' model to take into account an empirical medical observation: in some pathologies, gene carriers have a higher recurrence probability than non-carriers. Therefore probability distributions of events are not constant over time. We thus assume that insurance companies modify their prior beliefs on distribution probabilities from period to period. We consider two different settings: monopoly on the one hand; competition defined as the Stackelberg equilibrium of a game between an incumbent company being informed on the past morbid episodes of its own policyholders and acting as a leader, and an uninformed representative rival company acting as a follower. In the monopolistic setting we show that updating of beliefs basically prejudices to high-risk individuals that experience a morbid event in the first period. The reason is that in the second period, these policyholders cannot get full insurance as they do in the Cooper and Hayes' model, but have partial insurance with utility deterioration. On the opposite low-risk individuals always get partial insurance. Besides, all the policyholders are experience-rated. When entry of rival companies is assumed, high-risk individuals are no longer experience-rated. This situation is however beneficial for all policyholders but detrimental to the incumbent company which is compelled to give up its former monopoly profit to keep its clientele.

Our contribution is limited to a characterisation of the properties of separating equilibria. It remains to verify the existence of these equilibriums and their Pareto-improvement compared to pooling equilibria.

Several extensions of our model can be suggested. On the one hand, the robustness of our results could be assessed under different informational settings, mainly when policyholders also update their beliefs vis-à-vis morbid events distribution. On the other hand genetic tests are admittedly not fully informative: some tails entail substantial technical errors others interpretative errors. Eventually we have assumed that all potential subscribers take a genetic test before entering the health insurance market, but the implied insurance premiums are likely to deter certain categories of subscribers (whether based on types or on wealth constraints).
Appendix

Monopoly case

The program for agents experiencing a disease in the first period

The high-risk agents

Let $\mu_1$ and $\mu_2$ be the Lagrange multipliers respectively associated with the participation constraints for high- and low-risk agents experiencing a disease in the first period. From our assumption, the high-risk agents experiencing a loss in the first period have a higher loss probability in the second period. Thus, their participation constraint is always binding: in other words, $\mu_1 = 0$. On the other hand, the low-risk agents are indifferent between choosing the contract offered by the insurance company and opting to stay out of the market: $\mu_2 > 0$.

Let us denote $\epsilon_1$ and $\epsilon_2$ the multipliers associated with the self-selection constraints for high- and low risk agents.

**Assumption 1:** $\epsilon_1 > 0$ and $\epsilon_2 = 0$

In the second period, the high-risk agents having the disease during the first period strictly prefer their own contract to the contract designed low-risks. On the other hand, the low-risk agents strictly prefer their contract to the one offered to the high-risk agents.

**Assumption 2:** $\epsilon_1 = 0$ and $\epsilon_2 > 0$

If true, the individual rationality constraint for low-risk agents is not fulfilled.

Then only the first assumption is verified, i.e. $\epsilon_1 > 0; \epsilon_2 = 0$.

\[
\begin{align*}
N_{HS} &= \frac{\epsilon_1 P_H}{q_{HS}} u'(w - D + \beta_{HS}) \quad (a) \\
N_{HS} &= \frac{\epsilon_1 (1 - p_H)}{q_{HS}} u'(w - \alpha_{HS}) \quad (b)
\end{align*}
\]

From (a) and (b), we check whether
\[
\frac{u'(w - D + \beta_{HS})}{u'(w - \alpha_{HS})} = \frac{1 - p_H}{p_H} \frac{q_{HS}}{1 - q_{HS}} > 1.
\]

In other words, the optimal contract locates the agents at the intersection of the insurer profit line and their indifference curve. Since $q_{HS} > p_H$, we have $u'(w - D + \beta_{HS}) > u'(w - \alpha_{HS})$, i.e. $\beta_{HS} < D - \alpha_{HS}$. The high-risk agent with a loss in the first period obtains less than full insurance.

The optimal second period contract of low-risk agents

\[
\begin{align*}
N_{LS} &= \left[\frac{\mu_2 P_L}{q_{LS}} - \frac{\epsilon_1 P_H}{q_{LS}}\right] u'(w - D + \beta_{LS}) \quad (c) \\
N_{LS} &= \left[\frac{\mu_2 (1 - p_L)}{q_{LS}} - \frac{\epsilon_1 (1 - p_H)}{q_{LS}}\right] u'(w - \alpha_{LS}) \quad (d)
\end{align*}
\]

with these two relations, we get
\[
\frac{u'(w - D + \beta_{LS})}{u'(w - \alpha_{LS})} > 1 \quad \text{if and only if} \quad \frac{\epsilon_1}{\mu_2} > \frac{p_L - q_{LS}}{p_H - q_{LS}}.
\]

It means that, in the second period, the optimal contract of low-risk agents gives $\beta_{LS} < D - \alpha_{LS}$. In other words, in the second period the agents still get less than full insurance.
The program agents with no-loss in the first period

Following the notations of the previous program, let \( \mu_3 \) and \( \mu_4 \) be the Lagrange multipliers respectively associated with the participation constraints for high- and low-risk agents. The participation constraint for low-risk agents is binding, i.e. we assume that \( \mu_3 = 0 \) and \( \mu_4 > 0 \).

As regards the Lagrange multipliers \( \epsilon_3 \) and \( \epsilon_4 \) associated with the self-selection constraints, we assume that \( \epsilon_3 > 0 \) and \( \epsilon_4 = 0 \). Indeed, it is irrational to assume that the low-risk agents who do not have the disease in the first period choose the contract designed for high-risk agents, since \( p_H > p_L \). This assumption respects the individual rationality constraint for the low-risk agents.

The high-risk agents

\[
\begin{align*}
N_{HN} &= \epsilon_3 u'(w-D + \beta_{HN}) \\
N_{HN} &= \epsilon_3 u'(w-\alpha_{HN})
\end{align*}
\]

From (e) and (f) we have \( u'(w-D + \beta_{HN}) = u'(w-\alpha_{HN}) \). In other words, the high-risk agents with no-loss in the first period get full insurance in the second period \( \beta_{HN} = D - \alpha_{HN} \).

The low-risk agents

\[
\begin{align*}
N_{LN} &= \left[\frac{\mu_4 \cdot p_L}{q_{LN}} - \frac{\epsilon_3 \cdot p_H}{q_{LN}}\right] u'(w-D+\beta_{LN}) \\
N_{LN} &= \left[\frac{\mu_4 (1-p_L)}{1-q_{LN}} - \frac{\epsilon_3 (1-p_H)}{1-q_{LN}}\right] u'(w-\alpha_{LN})
\end{align*}
\]

From (g) and (h) we get \( \frac{u'(w-D+\beta_{LN})}{u'(w-\alpha_{LN})} > 1 \). This inequality is true if:

\[
\frac{\mu_4 \cdot p_L}{q_{LN}} - \frac{\epsilon_3 \cdot p_H}{q_{LN}} > \frac{\mu_4 (1-p_L)}{1-q_{LN}} - \frac{\epsilon_3 (1-p_H)}{1-q_{LN}}.
\]

It can easily shown that this is true since \( q_{LN} = p_L < p_H \). In conclusion, the optimal second-period contract for the low-risk agents with no loss in the first period is less than full insurance: \( \beta_{LN} < D - \alpha_{LN} \).

Competition case

Entrant

The second-period entrant tries to attract in his portfolio the agents of the incumbent company. However, the strategy of the incumbent will be to deter the entrant strategy thanks to cream-skimming constraints.

Let \( \epsilon_H \) be the Lagrange multiplier associated with the self-selection constraint for the high-risk agents. And let \( \epsilon_L \) be the Lagrange multiplier associated with self-selection constraint for the low-risk agents. We assume that \( \epsilon_H > 0 \) and \( \epsilon_L = 0 \), i.e. the high-risk agent is indifferent between his contract and the contract designed for the low-risk agent. However, the low-risk agent will strictly prefer his contract rather than the contract designed for the high-risk agent.

Let \( \gamma_H, \gamma_L > 0 \) be the multipliers respectively associated with the zero-profit constraints for high- and low-risk agent. The profit of the entrant is nil.

\[
\begin{align*}
\gamma_H &= \epsilon_H u'(w-D + \beta_H) \\
\gamma_H &= \epsilon_H u'(w-\alpha_H)
\end{align*}
\]
It is straightforward to show that $\bar{\beta}_{H} = D - \bar{\alpha}_{H}$, i.e. the entrant will offer full insurance to high-risk agents.

On the other hand, the first order condition for the low-risk type is:

$$
\begin{align*}
\gamma_{L} &= \left[1 - \epsilon_{H} \frac{p_{H}}{p_{L}} - \frac{1}{p_{L}}\right] u'(w - D + \beta_{L}) \\
\gamma_{L} &= \left[1 - \epsilon_{H} \frac{1 - p_{H}}{1 - p_{L}}\right] u'(w - \bar{\alpha}_{L})
\end{align*}
$$

The optimal contract for the low-risk agent is less than full insurance. Indeed, we verify that $u'(w - D + \beta_{L}) > 1$ since $p_{H} > p_{L}$.

The incumbent firm

We define the cream-skimming constraints as the new participation constraints. The strategy of the incumbent firm is now to deter the entrant strategy which is to attract its policyholders. We assume that $\mu_{5} > 0; \epsilon_{5} > 0; \eta_{5} > 0$.

The high-risk agents with no loss in the first period

$$
\begin{align*}
\gamma_{1} &= \frac{p_{H}}{q_{\eta_{5}N}} \left[1 + \mu_{5} + \epsilon_{5}\right] u'(w - D + \beta_{\eta_{5}N}) \\
\gamma_{1} &= \frac{1 - p_{H}}{1 - q_{\eta_{5}N}} \left[1 + \mu_{5} + \epsilon_{5}\right] u'(w - \alpha_{\eta_{5}N})
\end{align*}
$$

From these two relations, we have: $u'(w - D + \beta_{\eta_{5}N}) = u'(w - \alpha_{\eta_{5}N})$. Indeed, we must keep in mind that $q_{\eta_{5}N} = p_{H}$.

On the other hand, the slope of the high-risks indifference curve at the incumbent firm’s contract is equal to the slope of their indifference curve when they choose the contract supplied by the entrant. Furthermore, the contract locates the agent at the tangency of his indifference curve with the insurer’s profit line.

$$
MRS = \frac{1 - q_{\eta_{5}N}}{q_{\eta_{5}N}} \frac{u'(w - \alpha_{\eta_{5}N})}{u'(w - D + \beta_{\eta_{5}N})} = \frac{1 - p_{H}}{p_{H}} \frac{\left[1 + \mu_{5} + \epsilon_{5}\right]}{u'(w - D + \beta_{\eta_{5}N})} = \frac{1 - p_{H}}{p_{H}}
$$

The high-risk agents obtain full insurance whatever their company’s choice. The slopes of the two indifference curve are identical. Furthermore, the two curves and the insurer profit line are tangent. The slope of the insurer profit line is $1 - q_{\eta_{5}N} = \frac{1 - p_{H}}{p_{H}}$. Consequently, the no-loss high-risk agents will choose either the incumbent firm contract or the entrant contract.

The low-risk agents with no loss in the first period

$$
\begin{align*}
\gamma_{2} &= \left[\frac{p_{L}}{q_{\eta_{5}N}} + \mu_{6} + \frac{p_{L}}{q_{\eta_{5}N}} - \frac{1}{p_{L}} - \frac{1}{q_{\eta_{5}N}}\right] u'(w - D + \beta_{\eta_{5}N}) \\
\gamma_{2} &= \left[\frac{1 - p_{L}}{1 - q_{\eta_{5}N}} + \mu_{6} \frac{1 - p_{L}}{1 - q_{\eta_{5}N}} - \frac{1}{q_{\eta_{5}N}} - \frac{1 - p_{L}}{1 - q_{\eta_{5}N}}\right] u'(w - \alpha_{\eta_{5}N})
\end{align*}
$$

we have $u'(w - D + \beta_{\eta_{5}N}) > u'(w - \alpha_{\eta_{5}N})$ if and only if $1 + \mu_{6} - \epsilon_{5} \frac{1 - p_{H}}{1 - p_{L}} > 1 + \mu_{6} - \epsilon_{5} \frac{p_{H}}{p_{L}}$ because $q_{\eta_{5}N} = p_{H}$.

This relation is always verified since $p_{H} > p_{L}$. In equilibrium, the low-risks with no loss in the first period obtain less than full insurance, $\beta_{\eta_{5}N} < D - \alpha_{\eta_{5}N}$.
On the one hand, the cream-skimming constraint specifies that the agent is indifferent between the contract of the incumbent firm and the entrant contract, on the other hand since the optimal contract locates the agents at the intersection of the insurer profit line \( \frac{1-q_{LN}}{q_{LN}} = \frac{1-p_L}{p_L} \) and the agent indifference curve, we have in fact \((\alpha_{LN}, \beta_{LN}) = (\bar{\alpha}_L, \bar{\beta}_L)\).

**The high-risk with a loss in the first period**

Let us assume \( \mu_7 \geq 0; \epsilon_7 > 0; \epsilon_8 = 0 \). The variable \( \mu_7 \) is the Lagrange multipliers associated with the cream-skimming constraint:

\[
EU(\alpha_{HS}, \beta_{HS}, p_H) \geq EU(\bar{\alpha}_H, \bar{\beta}_H, p_H).
\]

**Assumption 1:** \( \mu_7 = 0 \)

The high-risk agent strictly prefers the contract supplied by the incumbent company to the one offered by the entrant. We denote by \( \gamma_3 > 0 \) and \( \epsilon_7 > 0 \) the Lagrange multipliers respectively associated with the zero-profit constraint and the self-selection constraint for the high-risk agents. The first order condition gives:

\[
\begin{align*}
\gamma_3 &= \frac{p_H}{q_{HS}}(1+\epsilon_7)u'(w-D+\beta_{HS}) \\
\gamma_3 &= \frac{1-p_H}{1-q_{HS}}(1+\epsilon_7)u'(w-\alpha_{HS})
\end{align*}
\]

if and only if \( u'(w-D+\beta_{HS}) > u'(w-\alpha_{HS}) \) i.e. \( \frac{1-p_H}{p_H} > \frac{1-q_{HS}}{q_{HS}} \). This relation is always fulfilled since \( q_{HS} > p_H \).

The optimal contract \((\alpha_{HS}, \beta_{HS})\) locates the agents at the intersection of their indifference curve and the zero profit line.

Indeed, we have:

\[
\frac{1-p_H}{p_H} \frac{u'(w-\alpha_{HS})}{u'(w-D+\beta_{HS})} = \frac{1-q_{HS}}{q_{HS}}.
\]

Given the entrant’s strategy, we only have to verify whether the contract supplied by the firm in a monopoly position in the first period is an optimal contract.

We know that the entrant offers \((\bar{\alpha}_H, \bar{\beta}_H)\) to all high-risk agents. This contract is located at the intersection of the indifference curve and the entrant’s zero-profit line:

\[
\frac{1-p_H}{p_H} = \frac{1-p_H}{p_H} \frac{u'(w-\bar{\alpha}_H)}{u'(w-D+\bar{\beta}_H)}
\]

In absolute value, the slope of the entrant zero profit line is greater than the slope of the incumbent company:

\[
\frac{1-p_H}{p_H} \frac{u'(w-\bar{\alpha}_H)}{u'(w-D+\bar{\beta}_H)} > \frac{1-q_{HS}}{q_{HS}}.
\]

In other words, the optimal contract offered by the entrant provides more utility than the one offered by the incumbent company. We can verify that the slope of the high-risk indifference curve when he chooses the competitive contract is greater than the slope of his indifference curve when he chooses the contract of the incumbent firm:

\[
\frac{1-p_H}{p_H} \frac{u'(w-D+\beta_{HS})}{u'(w-\alpha_{HS})} > \frac{1-q_{HS}}{q_{HS}} \frac{u'(w-D+\beta_{HS})}{u'(w-\alpha_{HS})}
\]

i.e., the high-risk agent with a loss will probably quit the incumbent company to the entrant in the second period. Indeed, the contract offered by the entrant will bring more satisfaction.

**Assumption 2:** Let us assume that the cream-skimming constraint is binding \( \mu_7 > 0 \).

The contract offered by the incumbent firm provides as utility as the contract offered by the entrant. In equilibrium, the contract offered by the incumbent firm is less than full insurance. This contract also breaks even.
The first order condition allows us to check that \( \frac{1-p_H}{p_H} \frac{u'(w-a_{HS})}{u'(w-D+\beta_{HS})} = \frac{1-q_{HS}}{q_{HS}} \). In other words, the optimal contract of the incumbent firm locates agents at the intersection of the indifference curve and the zero-profit line.

Otherwise, we verify that the first order condition gives:

\[
1-p_H \frac{u'(w-\alpha_H)}{u'(w-D+\beta_H)} = \frac{1-p_H}{p_H}
\]

when the high-risk agent chooses the entrant’s contract.

It means that the optimal contract of the entrant locates the agent at the tangency of his indifference curve with the zero profit line.

This situation challenges the hypothesis \( \mu_\gamma > 0 \). It can be verified that \( \frac{1-q_{HS}}{q_{HS}} < \frac{1-p_H}{p_H} \), i.e. the entrant zero profit-line is above the incumbent firm zero profit line. Consequently, the high-risk indifference curve could not be neither tangent at the entrant zero profit line, nor crossing the incumbent firm zero-profit line. The hypothesis \( \mu_\gamma > 0 \) is only compatible with \( \frac{1-q_{HS}}{q_{HS}} > \frac{1-p_H}{p_H} \), i.e. with a zero-profit line of the incumbent firm above the zero-profit line of the entrant.

Thus, \( \mu_\gamma \) is nil. It means that, in equilibrium, the contract offered by the incumbent firm is not possible.

The low-risk agent with a loss

\[
\gamma_4 = \left\{ \begin{array}{l}
\frac{p_L}{q_{LS}} + \mu_L \frac{p_L}{q_{LS}} - \epsilon_7 \frac{p_H}{q_{LS}} u'(w-D+\beta_{LS}) \\
\frac{1-p_L}{1-q_{LS}} + \mu_L \frac{1-p_L}{1-q_{LS}} - \epsilon_7 \frac{1-p_H}{1-q_{LS}} u'(w-\alpha_{LS})
\end{array} \right.
\]

The low-risk with a loss in the first period obtains partial insurance in the second period. We get \( \beta_{LS} < D - \alpha_{LS} \) if and only if:

\( \epsilon_7 (p_H - q_{LS}) > (p_L - q_{LS}) (1+\mu_L) \). The optimal contract locates the agents at the intersection of the zero-profit line of the incumbent firm and the low-risk indifference curve.
References


